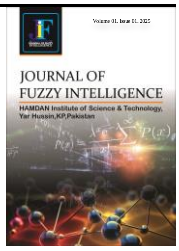
Journal of Fuzzy Intelligence

Journal homepage: www.mathfuzzyjournal.com

Vol:01 Issue: 01, 2025, pp. 20–30



A Novel Framework for Deep Neural Network Selection Based on Bipolar Fuzzy Power Weighted Aggregation Operators

Sanam Ayub[1] and Ijaz ur Rahman[2,*]

[1]Department of Mathematics, Riphah International University, Islamabad, Pakistan

[2]School of Management and Economics, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, P.R. China

[*]Corresponding author: ijazurrahman199@gmail.com

Available online: 24 December 2025

ABSTRACT

Bipolar fuzzy numbers are very suitable extension of fuzzy set theory and bipolar fuzzy number illustrate the uncertainty and vagueness. By applying the bipolar fuzzy numbers to a power aggregation operator to develop a bipolar fuzzy power aggregation operator and discuss their properties. We define distance measure between two bipolar fuzzy sets. We discuss the different properties of bipolar fuzzy aggregation operator. By using the bipolar fuzzy aggregation operator, we develop an algorithm for group decision making. A numerical example explains the proposed model of group decision making problem based on bipolar fuzzy aggregation operator. The proposed model apply to select the best deep neural network.

Keywords

Bipolar fuzzy sets; Bipolar fuzzy aggregation; Decision making problem; Deep neural network

1. Introduction

Fuzzy sets were introduced by Zadeh in 1965 [1] to direct the data and information acquiring none statically uncertainties. The concept of intuitionistic fuzzy set (IFS) [2] introduced by Atanassov has been established to be highly useful to deal with incorrectness/unreliability. IFS theory has been largely applied to areas like networking, artificial intelligence, soft decision making, operational research etc. One of the assuring role of IFS has been appeared in decision making problems [3-6]. In some real life positions, decision makers may not be able to exactly deliberate their perspective for the problems as they may not possess exact or enough level of knowledge of the problem or the decision makers are unable to disfavor express the degree to which one substitute are better than others in such cases, the decision maker may produce their choices for substitutes to a undoubting degree, but it is thinkable that they are not so assure about it. Thus it is very convenient to convey the decision maker desire values with the use of intuitionistic fuzzy values instead of exact numerical values. The aim of this research work is promote the bipolar fuzzy decision making model to selection of different alternatives based on multiple criteria's. A decision maker (expert) is frequently confronted the difficult of selecting a solution of a given set of finite numbers of alternative. The selected alternative is the top one or conciliation preference that satisfied definite predefined objectives. The techniques of multiple criteria decision making (MCDM) supports management and engineering decision in calculating and/or choosing the required one from finite number of alternatives, which are categorized by multiple criteria's. By using the vague sets, [7] given different method to solve multiple criteria/attribute decision making problems. Vague set is an intuitionistic fuzzy set.

The aggregation information operator is the remarkable and significant research area in AIFS theory that has been getting progressively in recent years. In the intuitionistic fuzzy information aggregation is the utmost imperative problem by using the binary operations to convey the combination. Atanassov introduced the elementary operations. In intuitionistic fuzzy information aggregation, there is another most important problem to combine a collection of intuitionistic fuzzy numbers/values into one intuitionistic fuzzy value and the algebraic operation laws of IFS play a vital role in aggregation operators. Many authors have been handled this type of problems in different techniques by using the binary operational laws [8, 9]. In [10, 11], Xu used the algebraic operational laws of IFSs to developed many aggregation operators for combining the IFVs and Xu and Yager constructed different fundamental geometric aggregation (GA) operators and given applications of GA to MADM by using intuitionistic fuzzy information's. In [12], Wei presented various IFS induced geometric aggregation operators and studied application of induced geometric aggregation operator in group decision making. Li [13] studied the extended version of the proposed (GOWA) operators by Yager [14]. Zhao et al. constructed new extended version of generalized aggregation operators i.e., (GIFWA) operator, generalized intuitionistic fuzzy ordered weighted averaging (GIFOWA) operator, and generalized intuitionistic fuzzy hybrid averaging operator. In [15], He et al., introduced intuitionistic fuzzy geometric interaction averaging operators and they applied to MCDM. In [16], Abdullah and Amin used the intuitionistic fuzzy decision making problem to S-box image encryptions.

Zhang started a new theory in fuzzy set theory, which is called a bipolar fuzzy set (BFS) [17, 18]. The structure of bipolar fuzzy set (BFS) is an extension of a fuzzy set (FS). The degree of membership of a bipolar fuzzy set is larger than a fuzzy set and an intuitionistic fuzzy set. i.e. the range of membership degree of bipolar fuzzy set is [-1,1]. The structure of a bipolar fuzzy set and an intuitionistic fuzzy set are different by the range of membership degree. The range of membership degree of an intuitionistic fuzzy set is [0, 1], while the bipolar fuzzy set is [-1,1]. The meaning of 0 membership degree in intuitionistic fuzzy set is an element that does not belong to a set while in bipolar fuzzy set means that the element is irrelevant to the corresponding property, in bipolar fuzzy set and intuitionistic fuzzy set, the membership degree (0,1] of an element represent that the element somewhat hold the property, and in bipolar fuzzy set the membership degree [-1,0) of an element represent the element somewhat hold the implicit counter property, while in intuitionistic fuzzy set this membership is not given. In various domains, it is significant to be able to deal with bipolar information. It is noted that positive information indicates what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. I. Bloch applied the concept of bipolar fuzzy set to mathematical morphology and different operations defined of bipolar fuzzy erosion and dilation. Applications of bipolar fuzzy sets in different field studied in [19-21]. Akram applied bipolar fuzzy sets in graph theory and he defined bipolar fuzzy graphs [22]. Further applications of bipolar fuzzy set in graph theory can be found in [23-27]. Recently, Gul, defined bipolar fuzzy aggregation operators and studied different properties. He defined different types of bipolar fuzzy aggregations operator’s i.e. bipolar fuzzy averaging weighted aggregation operators and bipolar fuzzy geometric aggregations operators

The aim of this article to promote the research in the direction of aggregation operators based on bipolar fuzzy information. Bipolar fuzzy numbers are very suitable extension of fuzzy set theory and bipolar fuzzy number illustrate very deeply the uncertainty and vagueness. By applying the bipolar fuzzy numbers to a power aggregation operator to develop a bipolar fuzzy power aggregation operator and discuss their properties. We define distance measure between two bipolar fuzzy sets. We discuss the different properties of bipolar fuzzy aggregation operator. By using the bipolar fuzzy aggregation operator, we develop an algorithm for group decision making. A numerical example explain the proposed model of group decision making problem based on bipolar fuzzy aggregation operator.

2. *Preliminaries*

This section provides the basic concept and fundamental result for the remaining paper.

2.1. *Power Aggregation Operator*

In this section we recall the concept of power aggregation operation, which is a process that combines different data of resources to a single information sources by using aggregation operator. The power aggregation operator to aggregate a correction of information data bj $(j = 1, 2, \dots, n)$. The power aggregate operator defined as following

$$PA(a_1,a_2,a_3,\dots,a_n) = \frac{\sum_{j=1}^{n}(1+T(a_j))a_j}{\sum_{j=1}^{n}(1+T(a_j)}$$

Where $T_{(aj)} = \sum_{j=1}^{n}\sup(a_j, a_j)$. The support for a_j from a_j and satisfied the support has the following.

(1) $Sup(a_i, a_j) \in [0,1]$

(2) $Sup(a_i, a_j) = Sup(a_j, a_i)$

(3) $Sup(a_i, a_j) \geq Sup(a_s, a_t)$ $if \left|a_i - a_j\right| < \left|a_s - a_t\right|$

2.2. *Bipolar fuzzy sets*

In this section we review the definitions of bipolar fuzzy sets and their basic operations.

Definition 1: Let $B_i = (\mu_i^+, \mu_i^-)$ $(i = 1,2)$ be any two BFNs, then

1: $B_1 \oplus B_2 = (\mu_1^+ + \mu_2^+ - \mu_1^+\mu_2^+, -|\mu_1^-||\mu_2^-|)$

2: $B_1 \otimes B_2 = (\mu_1^+ \mu_2^+, \mu_1^- + \mu_2^- + |\mu_1^-||\mu_2^-|$

3: $\lambda B_1 = (1-(1-\mu_1^+)^\lambda, -|\mu_1^-|, (1-\mu_1^+)^\lambda + |\mu_1^-|^\lambda),\ \lambda > 0$

4: $B_1^\lambda = (\mu_1^{+\lambda}, 1-(1-\mu_1^-)^\lambda, (1-\mu_1^-)^\lambda - \mu_1^{+\lambda}),\ \lambda > 0$

From above operations all the results are also BFNs and the following are all right.

1: If $\lambda_1 > \lambda_2$, then $\lambda_1 B \geq \lambda_2 B, B^{1-\lambda_1} \geq B^{1-\lambda_2}$, $0 < \lambda_1, \lambda_2 \leq 1$;

2: If $\mu_1^+ \geq \mu_2^+, \mu_1^- \leq \mu_2^-$, then $\lambda B_1 \geq \lambda B_2, B_1^\lambda \geq B_2^\lambda$, $0 < \lambda \leq 1$;

3: If $\mu_1^+ \geq \mu_3^+, \mu_2^+ \geq \mu_4^+, \mu_1^- \leq \mu_3^-, \mu_2^- \leq \mu_4^-$ then $B_1 \oplus B_3 \geq B_2 \oplus B_4, B_1 \otimes B_3 \geq B_2 \otimes B_4$

In fact, if $\lambda_1 > \lambda_2$, then $1-(1-\mu_1^+)^{\lambda_1} \geq 1-(1-\mu_1^+)^{\lambda_2}$ and $\mu_1^{-\lambda_1} \leq \mu_2^{-\lambda_2}$, therefore, $1-(1-\mu_1^+)^{\lambda_1} + |\mu_1^{-\lambda_1}| \geq 1-(1-\mu_1^+)^{\lambda_2} + |\mu_2^{-\lambda_2}|$ which implies $\lambda_1 B \geq \lambda_2 B$; if $\mu_1^+ \geq \mu_2^+, \mu_1^- \leq \mu_2^-$, then $1-(1-\mu_1^+)^\lambda + |\mu_1^{-\lambda}| \geq 1-(1-\mu_2^+)^\lambda + |\mu_2^{-\lambda}|$ and $\mu_1^{+\lambda} - (1-(1-\mu_1^-)^\lambda) \geq \mu_2^{+\lambda} - (1-(1-\mu_2^-)^\lambda)$,

Thus, $\lambda B_1 \geq \lambda B_2$ and $B_1^\lambda \geq B_2^\lambda$; if $\mu_1^+ \geq \mu_3^+, \mu_2^+ \geq \mu_4^+, \mu_1^- \leq \mu_3^-, \mu_2^- \leq \mu_4^-$, then

$$(\mu_1^+ + \mu_2^+ - \mu_1^+\mu_2^+ + |\mu_1^-||\mu_2^-|) = 1-(1-\mu_1^+)(1-\mu_2^+) + |\mu_1^-||\mu_2^-| \geq 1-(1-\mu_3^+)(1-\mu_4^+) + |\mu_3^-||\mu_4^-| = \mu_3^+ + \mu_4^+ - \mu_3^+\mu_4^+ + |\mu_3^-||\mu_4^-|$$

And $\mu_1^+\mu_2^+ - (\mu_1^- + \mu_2^- + |\mu_1^-||\mu_2^-|) = \mu_1^+\mu_2^+ - 1 + (1-\mu_1^-)(1-\mu_2^-)$

$$\geq \mu_3^+\mu_4^+ - 1 + (1-\mu_3^-)(1-\mu_4^-) = \mu_1^+\mu_2^+ - (\mu_3^- + \mu_4^- + |\mu_3^-||\mu_4^-|$$

Thus $B_1 \oplus B_3 \geq B_2 \oplus B_4, B_1 \otimes B_3 \geq B_2 \otimes B_4$

Definition 2: Let B_1 and B_2 be two bipolar fuzzy sets of X, then Hamming distance between B_1 and B_2 is defined as following.

$$d(\mathrm{B}_1, \mathrm{B}_2) = \frac{1}{2}\sum_{x \varepsilon X} \left|\mu_1^+(x) - \mu_2^+(x)\right| + \left|\mu_1^-(x) - \mu_2^-(x)\right|$$

3. *Bipolar Fuzzy Power Aggregation Operator*

In this section we realize the concept of power aggregation operation which is defined by Yager [14] aggregation information is a process that combines different data of resources to a single information source by using aggregation operator. The power aggregation operator to aggregate a correction of information data, $b_j = (j = 1,2,...,n)$. The bipolar fuzzy power aggregation operator defined as; Let $B_i = (\mu_i^+, \mu_i^-),\ i = 1,2,.......n$ be a family of BFNs, and the weight vector of B_i be denoted by $W = (w_1, w_2, ..., w_n)^T$, where $w_i \geq 0,\ i = 1,2,...,n$ and $\sum_{i=1}^n w_i = 1$. Then bipolar fuzzy power weighted average (BFPWA) operator can be define as follows.

$$BFPWA(B_1, B_2, ..., B_n) = \frac{(w_1(1+T(B_1))B_1) \oplus (w_2(1+T(B_2))B_2) \oplus ... \oplus (w_n(1+T(B_n))B_n)}{\sum_{i=1}^n w_i(1+T(B_i))}$$

By definition above the equation can be transformed in to the following form by using mathematical induction n.

$$BFPWA(B_1, B_2, ...B_n) = \left(\begin{array}{c} 1 - \prod_{j=1}^n (1-\mu_i^+)^{\frac{w_j(1+T(B_j))}{\sum_{i=1}^n w_i(1+T(B_i))}}, \\ \prod_{j=1}^n |\mu_i^-|^{\frac{w_j(1+T(B_j))}{\sum_{i=1}^n w_i(1+T(B_i))}} \end{array} \right),$$

where $T(B_i) = \sum_{\substack{j=1 \\ j \neq 1}}^n w_j Sup(B_i, B_j)$

and $Sup(B_i, B_j)$ is the support for B_i from B_j with the following conditions.

(1) $Sup(B_i, B_j) \in [0,1]$

(2) $Sup(B_i, B_j) = Sup(B_i, B_j)$

(3) $Sup(B_i, B_j) \geq Sup(B_s, B_t)$ if $d\left|B_i - B_j\right| < \left|B_s - B_t\right|$

where d is a distance measure such as the normalized Hamming distance or the normalized Euclidean distance. Especially, if $W = (1/n, 1/n, ... 1/n)^T$, then the $BFPWA$ operator defined above reduces to a bipolar fuzzy power average ($BFPA$) operator.

$$BFPA(B_1, B_2, ..., B_n) = \frac{\begin{pmatrix} (1+T(B_1))B_1) \oplus ((1+T(B_2))B_2) \oplus \\ ... \oplus ((1+T(B_n))B_n) \end{pmatrix}}{\sum_{i=1}^{n}(1+T(B_i))}$$

$$= \left(1-\prod_{j=1}^{n}(1-\mu_i^{+})^{\frac{(1+T(B_j))}{\sum_{i=1}^{n}(1+T(B_i))}}, \prod_{j=1}^{n}|\mu_i^{-}|^{\frac{(1+T(B_j))}{\sum_{i=1}^{n}(1+T(B_i))}} \right)$$

Where $T(B_i) = \frac{1}{n}\sum_{\substack{j=1 \\ j\neq 1}}^{n} Sup(B_i, B_j)$

It can be easily proved that the $BFPWA$ operator has the following desirable properties.

Theorem 1: (commutative) Let $(B_1, B_2, ..., B_n)$ be a vector of n $BFNs$, and $(B_1', B_2', ..., B_n')$ be any permutation of $(B_1, B_2, ..., B_n)$. Then,

$$BFPWA(B_1, B_2, ..., B_n) = BFPWA(B_1', B_2', ..., B_n')$$

Theorem 2: (Idempotency) Let $B_j\,(j = 1,2,....n)$ be a family of $BFNs$, if $B_j = B$, for all j, then

$$BFPWA(B_1, B_2, ...B_n) = B$$

Theorem 3: (Boundedness) Let $B_j\,(j = 1,2,....n)$ be a family of $BFNs.$ Then,

$$B_{\min} \leq BFPWA(B_1, B_2, ...B_n) \leq B_{\max}$$

Where $B_{\min} = (\min_j\{\mu_j^{+}\}, \max_j\{\mu_j^{-}\})$ and $B_{\max} = (\max_j\{\mu_j^{+}\}, \min_j\{\mu_j^{-}\})$ based on the $BFPWA$ operator as defined above and the geometric mean, here we define an bipolar fuzzy power weighted geometric $(BFPWG)$ operator.

$$BFPWG(B_1, B_2, ...B_n) = (B_1)^{\frac{1}{n-1}\left(1-\frac{w_1(1+T(B_1))}{\sum_{i=1}^{n} w_i(1+T(B_i))}\right)} \otimes (B_2)^{\frac{1}{n-1}\left(1-\frac{w_2(1+T(B_2))}{\sum_{i=1}^{n} w_i(1+T(B_i))}\right)} \otimes ...... \otimes (B_n)^{\frac{1}{n-1}\left(1-\frac{w_n(1+T(B_n))}{\sum_{i=1}^{n} w_i(1+T(B_i))}\right)}$$

which can be transformed in to the following form by using mathematical induction on n

$$BFPWG(B_1, B_2, ...B_n) = \left(\prod_{j=1}^{n}(\mu_i^{+})^{\frac{1}{n-1}\left((1-\frac{w_j(1+T(B_j))}{\sum_{i=1}^{n} w_i(1+T(B_j))}\right)}, -1+\prod_{j=1}^{n}|-1+\mu_j^{-}|^{\frac{1}{n-1}\left((1-\frac{w_j(1+T(B_j))}{\sum_{i=1}^{n} w_i(1+T(B_j))}\right)} \right)$$

with the condition. Especially, if $W = (1/n, 1/n, ..., 1/n)^T$, then the $BFPWG$ operator reduces to a bipolar fuzzy power geometric ($BFPG$) operator.

$$BFPG(B_1,B_2,....B_n)=(B_1)^{\frac{1}{n-1}\left(1-\frac{(1+T(B_1))}{\sum_{i=1}^{n}(1+T(B_j))}\right)}\otimes(B_2)^{\frac{1}{n-1}\left(1-\frac{(1+T(B_2))}{\sum_{i=1}^{n}(1+T(B_j))}\right)}\otimes....\otimes(B_n)^{\frac{1}{n-1}\left(1-\frac{(1+T(B_n))}{\sum_{i=1}^{n}(1+T(B_j))}\right)}$$

$$=\left(\begin{array}{c}\prod_{j=1}^{n}(\mu_i^+)^{\frac{1}{n-1}\left(1-\frac{(1+T(B_j))}{\sum_{i=1}^{n}(1+T(B_j))}\right)},\\ -1+\prod_{j=1}^{n}|-1+\mu_j^-|^{\frac{1}{n-1}\left(1-\frac{(1+T(B_j))}{\sum_{i=1}^{n}(1+T(B_j))}\right)}\end{array}\right)$$

Similar to the $BFPWA$ operator and the $BFPWG$ operator has the following three properties

Theorem 4: Let ($B_1,B_2,...,B_n$) be a vector of n $BFNs$, and $(B'_1,B'_2,..,B'_n)$ be any permutation of $(B_1,B_2,...,B_n)$, then

1) (Commutative):

$$BFPWG\,(B_1,B_2,...,B_n)=BFPWG\,(B'_1,B'_2,..,B'_n)$$

2) (Idempotency): Let $B_j\,(j=1,2,....n)$ be a collection of $BFNs$, if $B_j=B$, for all j. Then

$$BFPWG\,(B_1,B_2,...,B_n)=B$$

3) (Boundedness): Let $B_j\,(j=1,2,....n)$ be a collection of $BFNs.$ Then,

$$B_{\min}\le BFPWG\,(B_1,B_2,...,B_n)\le B_{\max}$$

where $B_{\min}=(\min{}_j\{\mu_j^+\},\max{}_j\{\mu_j^-\})$ and $B_{\max}=(\max{}_j\{\mu_j^+\},\min{}_j\{\mu_j^-\})$

4. *Bipolar Fuzzy Group Decision Making based on Bipolar Fuzzy power Aggregation Operator*

In this section we use the bipolar fuzzy aggregation operator to bipolar fuzzy group decision making problem. We first develop a general algorithm for group decision making problem based on bipolar fuzzy information's. The algorithm will consist eight step and finally we derived our result based on closeness co-efficient.

Let $R^{(k)}=(\mathrm{B}_{ij}^{(k)})_{n*m}$ be a bipolar fuzzy decision matrix, where $\mathrm{B}_{ij}^{(k)}=\left\langle\mu_{ij}^{+(k)},\mu_{ij}^{-(k)}\right\rangle$ is a criteria which is given by expert k and denoted by BFV. Where μ_{ij}^- is denoted the negative information of each alternative based on each criteria c_j . Now we use the BFPWA operator to develop multi- criteria group decision making with bipolar fuzzy information. We develop a general algorithm for this group decision making. This algorithm will construct the following steps.

Step 1

Calculate the

$$\sup(\mathrm{B}_{ij}^{(k)},\mathrm{B}_{ij}^{(l)})=1-d(\mathrm{B}_{ij}^{(k)},\mathrm{B}_{ij}^{(l)}),\,k,l=1,2,3,...s$$

Which satisfy the support conditions. We use the normalized Hamming distance

$$d(r_{ij}^{(k)},r_{ij}^{(l)})=\frac{1}{2}\left(\left|\mu_{ij}^{+(k)}-\mu_{ij}^{+(l)}\right|+\left|\mu_{ij}^{-(k)}-\mu_{ij}^{-(l)}\right|\right),$$
$$k,l=1,2,3,..s$$

Step 2: In this step, we use the known weights (λ_k $(k=1,2,3,...,s)$ e_k $(k=1,2,3,....s)$ to find out the weighted support $T(\mathrm{B}_{ij}^{(k)})$ of BFV $\mathrm{B}_{ij}^{(k)}$ by the other BFV $\mathrm{B}_{ij}^{(l)}$ $(l=1,2,3,...,s)$ $(k\neq l)$

$$T(\mathrm{B}_{ij}^{(k)})=\sum_{\substack{l=1\\ L\neq K}}^{s}\lambda_l\sup(\mathrm{B}_{ij}^{(k)},\mathrm{B}_{ij}^{(l)})$$

Step 3: In this step, we calculate the weights associated with bipolar fuzzy variables. $\mathrm{B}_{ij}^{(k)}\,(k=1,2,3,.....s)$.

$$E_{ij}^{(k)}=\frac{\lambda_k(1+T(\mathrm{B}_{ij}^{(k)})}{\sum_{k=1}^{s}\lambda_k(1+T(\mathrm{B}_{ij}^{k})},\;k=1,2,3,...s$$

where $E_{ij}^{(k)}\ge 0\,(\mathrm{k}=1,2,3,...,\mathrm{s})$

Step 4: In this step we apply the bipolar fuzzy power average operator to compose opinion of each expert to single decision matrix.

$$B_{ij} = BFPWA\ (B_{ij}^{(1)}, B_{ij}^{(2)}, \ldots\ldots B_{ij}^{(k)})$$
$$= (1-\prod_{k=1}^{s}(1-\mu_{ij}^{+(k)})^{E_{ij}^{(k)}}, -\prod_{k=1}^{s}\left|\mu_{ij}^{-(k)}\right|^{E_{ij}^{(k)}})$$

Step 5: In this step we find the positive ideal solution and negative ideal solution. The positive ideal solution is denoted by P and defined as following;

$$P = \left((\mu_1^+, \mu_1^-)^+, (\mu_2^+, \mu_2^-)^+, \ldots, (\mu_m^+, \mu_m^-)^+\right)$$

Negative ideal solution is denoted by N and defined as following;

$$N = \left((\mu_1^+, \mu_1^-)^-, (\mu_2^+, \mu_2^-)^-, \ldots, (\mu_m^+, \mu_m^-)^-\right)$$

where

$$(\mu_j^+, \mu_j^-)^+ = (\max \mu_{ij}^+,\ \min \mu_{ij}^-)$$
$$(\mu_j^+, \mu_j^-)^- = (\min \mu_{ij}^+,\ \max \mu_{ij}^-)$$

Step 6: In this section we calculate the distance measure between alternatives and positive ideal solution and negative ideal solution

$$d(B_1, B_2) = \frac{1}{2}\sum_{x \varepsilon X}\left|\mu_1^+(x) - \mu_2^+(x)\right| + \left|\mu_1^-(x) - \mu_2^-(x)\right|$$

Step 7: In this step we calculate the relative closeness co-efficient of each alternative with distance measure between alternatives and positive ideals solution, and negative ideals solution.

$$C(B_1) = \frac{d(B_1, P)}{d(B_1, P) + d(B_1, N)}$$

Step 8: Find the optimal values, the maximum value of the relative closeness co-efficient.

5. Problem statement

Deep neural networks (DNNs) are essential in modern AI for addressing intricate classification and forecasting challenges. Nevertheless, determining the right network architecture or model configuration involves testing numerous options against a variety of evaluation metrics. The selection process can be difficult because of the UCI in the selection of alternatives, the mixed and competing priorities, and the subjective nature of the evaluation process. To resolve this, we implement a new decision-making paradigm that combines fuzzy logic with evaluation of deep neural networks. The goal is to rank alternatives and select the best one according to a multi-criteria decision-making model. The current case study provides five potential DNN alternatives and five evaluation DNN structures and criteria:

- Convolutional Neural Network (CNN) with 3 layers-(B_1)
- CNN with 5 layers-(B_2)
- Fully Connected Neural Network (FCNN) -(B_3)
- Recurrent Neural Network (RNN)-(B_4)
- Hybrid CNN-RNN model-(B_5)

In this study, we apply the proposed decision-making method to select the best neural network model from five alternatives. The selection process is based on five key criteria:

- Accuracy-(c_1)
- Training Time-(c_2)
- Robustness-(c_3)
- Scalability-(c_4)
- Complexity-(c_5)

6. Numerical Applications in Decision Making Problem

In order to illustrate efficiency and practical advantages of the proposed procedure, we consider a real example of a company to select a candidate for a vacant post. The company received five applications for vacant post and let B_1, B_2, B_3, B_4, B_5 be five applications of the applicant and the company have three expert to select the best applicant based on set of criteria's $c_1 = Previous\ Experience, c_2 = Academics\ marks, c_3 = Interiev\ marks, c_4 = Personality\ , c_5 = Commuications\ skills$. The weight of each experts are given as $\lambda = (0.25, 0.35, 0.4)$. After the interview of the applicants, the experts construct three bipolar fuzzy decision matrix as shown in Table 1, Table 2, Table 3.

Table 1: Bipolar Fuzzy Decision Matrix of Expert 1

B_i	c_1	c_2	c_3	c_4	c_5
B_1	$\langle 0.2,-0.5\rangle$	$\langle 0.3,-0.7\rangle$	$\langle 0.4,-0.8\rangle$	$\langle 0.1,-0.3\rangle$	$\langle 0.2,-0.5\rangle$
B_2	$\langle 0.4,-0.6\rangle$	$\langle 0.4,-0.9\rangle$	$\langle 0.1,-0.3\rangle$	$\langle 0.3,-0.4\rangle$	$\langle 0.1,0.9\rangle$
B_3	$\langle 0.3,-0.8\rangle$	$\langle 0.5,-0.3\rangle$	$\langle 0.2,-0.6\rangle$	$\langle 0.5,-0.5\rangle$	$\langle 0.8,-0.8\rangle$
B_4	$\langle 0.7,-0.9\rangle$	$\langle 0.5,-0.5\rangle$	$\langle 0.5,-0.6\rangle$	$\langle 0.2,-0.9\rangle$	$\langle 0.3,-0.7\rangle$
B_5	$\langle 0.8,-0.3\rangle$	$\langle 0.6,-0.9\rangle$	$\langle 0.8,-0.9\rangle$	$\langle 0.6,-0.8\rangle$	$\langle 0.8,-0.8\rangle$

Table 2: Bipolar Fuzzy Decision Matrix of Expert 2

B_i	c_1	c_2	c_3	c_4	c_5
B_1	$\langle 0.3,-0.6\rangle$	$\langle 0.2,-0.3\rangle$	$\langle 0.3,-0.2\rangle$	$\langle 0.9,-0.6\rangle$	$\langle 0.3,-0.1\rangle$
B_2	$\langle 0.5,-0.8\rangle$	$\langle 0.3,-0.7\rangle$	$\langle 0.2,-0.6\rangle$	$\langle 0.4,-0.8\rangle$	$\langle 0.6,-0.2\rangle$
B_3	$\langle 0.4,-0.9\rangle$	$\langle 0.2,-0.6\rangle$	$\langle 0.8,-0.9\rangle$	$\langle 0.3,-0.2\rangle$	$\langle 0.9,-0.3\rangle$
B_4	$\langle 0.3,-0.6\rangle$	$\langle 0.1,-0.9\rangle$	$\langle 0.3,-0.3\rangle$	$\langle 0.1,-0.8\rangle$	$\langle 0.4,-0.4\rangle$
B_5	$\langle 0.1,-0.1\rangle$	$\langle 0.1,-0.6\rangle$	$\langle 0.4,-0.1\rangle$	$\langle 0.5,-0.1\rangle$	$\langle 0.8,-0.5\rangle$

Table 3: Bipolar Fuzzy Decision Matrix of Expert 3

B_i	c_1	c_2	c_3	c_4	c_5
B_1	$\langle 0.1,-0.8\rangle$	$\langle 0.4,-0.8\rangle$	$\langle 0.2,-0.1\rangle$	$\langle 0.8,-0.6\rangle$	$\langle 0.1,-0.8\rangle$
B_2	$\langle 0.3,-0.8\rangle$	$\langle 0.1,-0.5\rangle$	$\langle 0.1,-0.9\rangle$	$\langle 0.3,-0.5\rangle$	$\langle 0.5,-0.1\rangle$
B_3	$\langle 0.2,-0.9\rangle$	$\langle 0.3,-0.9\rangle$	$\langle 0.7,-0.7\rangle$	$\langle 0.4,-0.1\rangle$	$\langle 0.4,-0.4\rangle$
B_4	$\langle 0.1,-0.1\rangle$	$\langle 0.4,-0.8\rangle$	$\langle 0.2,-0.9\rangle$	$\langle 0.7,-0.7\rangle$	$\langle 0.2,-0.8\rangle$
B_5	$\langle 0.8,-0.9\rangle$	$\langle 0.2,-0.6\rangle$	$\langle 0.8,-0.3\rangle$	$\langle 0.4,-0.9\rangle$	$\langle 0.1,-0.9\rangle$

We use equations $T(B_{ij}^{(k)}) = \sum_{\substack{l=1 \\ L\neq K}}^{s} \lambda_l \sup(B_{ij}^{(k)}, B_{ij}^{(l)})$ to calculate the weighted support of each alternatives using know weight of each expert. The weighted support of each alternative of each expert is given in Table 4, 5 & 6

Table: 4 Weighted Support of Expert 1

$T(B_{ij}^{(k)})$	c_1	c_2	c_3	c_4	c_5
B_1	0.635	0.623	0.447	0.357	0.582
B_2	0.637	0.557	0.56	0.642	0.300
B_3	0.675	0.485	0.472	0.562	0.485
B_4	0.347	0.530	0.542	0.575	0.640
B_5	0.472	0.470	0.420	0.550	0.365

Table 5 Weighted Support of Expert 2

$T(B_{ij}^{(k)})$	c_1	c_2	c_3	c_4	c_5
B_1	0.545	0.447	0.522	0.492	0.407
B_2	0.572	0.532	0.520	0.507	0.460
B_3	0.585	0.495	0.477	0.547	0.455
B_4	0.422	0.470	0.447	0.485	0.500
B_5	0.237	0.530	0.380	0.370	0.255

Table 6 Weighted Support of Expert 3

$T(B_{ij}^{(k)})$	c_1	c_2	c_3	c_4	c_5
B_1	0.480	0.452	0.452	0.457	0.392
B_2	0.527	0.442	0.455	0.517	0.415
B_3	0.540	0.430	0.472	0.502	0.395
B_4	0.302	0.480	0.402	0.390	0.487
B_5	0.262	0.495	0.420	0.457	0.320

Now we calculate weights associated with bipolar fuzzy variables $B_{ij}^{(k)}\,(k = 1,2,3,....s)$. The weights associated is denoted by

$$E_{ij}^{(k)} = \frac{\lambda_k (1 + T(B_{ij}^{(k)})}{\sum_{k=1}^{s} \lambda_k (1 + T(B_{ij}^{k})}, \; k = 1,2,3,...s$$

The values of weights associated are in Table 7, 8 & 9.

Table: 7 Weights associated of Expert 1

$E_{ij}^{(k)}$	c_1	c_2	c_3	c_4	c_5
B_1	0.264	0.271	0.244	0.234	0.273
B_2	0.260	0.259	0.259	0.265	0.231
B_3	0.262	0.252	0.249	0.254	0.257
B_4	0.247	0.256	0.265	0.267	0.268
B_5	0.281	0.256	0.252	0.267	0.260

Table: 8 Weights associated of Expert 2

$E_{ij}^{(k)}$	c_1	c_2	c_3	c_4	c_5
B_1	0.350	0.338	0.360	0.361	0.340
B_2	0.350	0.356	0.353	0.341	0.364
B_3	0.348	0.356	0.350	0.352	0.353
B_4	0.366	0.345	0.348	0.353	0.343
B_5	0.330	0.356	0.343	0.330	0.335

Table: 9 Weights associated of Expert 3

$E_{ij}^{(k)}$	c_1	c_2	c_3	c_4	c_5
B_1	0.384	0.388	0.393	0.403	0.385
B_2	0.388	0.383	0.386	0.392	0.403
B_3	0.387	0.390	0.399	0.391	0.387
B_4	0.383	0.397	0.385	0.378	0.388
B_5	0.386	0.398	0.403	0.575	0.403

Now we aggregate bipolar fuzzy values of all experts' opinion of alternative by using the bipolar fuzzy power aggregation operator. The aggregated bipolar fuzzy decision matrix is given in Table 10.

Table 10: Aggregated bipolar fuzzy decision matrix

$\langle \mathbf{0.203}, -\mathbf{0.637} \rangle$	$\langle \mathbf{0.311}, -\mathbf{0.553} \rangle$	$\langle \mathbf{0.29}, -\mathbf{0.214} \rangle$	$\langle \mathbf{0.783}, -\mathbf{0.509} \rangle$	$\langle \mathbf{0.202}, \mathbf{0.346} \rangle$
$\langle \mathbf{0.404}, -\mathbf{0.741} \rangle$	$\langle 0.26, -0.655 \rangle$	$\langle 0.137, -0.586 \rangle$	$\langle 0.337\ , -0.553 \rangle$	$\langle 0.473, -0214 \rangle$
$\langle \mathbf{0.302}, -\mathbf{0.871} \rangle$	$\langle 0.327, -0.589 \rangle$	$\langle 0.668, -0.734 \rangle$	$\langle 0.396, -0.192 \rangle$	$\langle 0.76, -0.432 \rangle$
$\langle \mathbf{0.376}, -\mathbf{0.333} \rangle$	$\langle 0.342, -0.738 \rangle$	$\langle 0.327, -0.550 \rangle$	$\langle 0.425, -0.784 \rangle$	$\langle 0.302, -0.607 \rangle$
$\langle \mathbf{0.671}, -\mathbf{0.319} \rangle$	$\langle 0.304, -0.661 \rangle$	$\langle 0.709, -0.271 \rangle$	$\langle 0.537, -0.413 \rangle$	$0.592, -0.715$

Now we find positive ideal solution and negative ideal solution from table 10 by using equations (5) and (6) respectively.

$$P = \{(0.671,-0.871), (0.342,-0.738), (0.709,-0.734), (0.783,-0.784), (0.76,-0.715)\}$$

$$N = \{(0.203,-0.319), (0.26,-0.553), (0.137,-0.214), (0.337,-0.192), (0.202,-0.214)\}$$

The Hamming distance measure formula used to find distance measure between each alternatives and PIS and NIS, respectively. The distance measure of each alternative from PIS and NIS are shown in table 11. Graphically representation of the distance measure of each alternative from PIS and NIS are shown in Figure 1.

Table 11: Distance measure of each alternative from PIS and NIS

Distance of PIS	Distance of NIS
$\boldsymbol{d(B_1, P)}$=2.0825	$d(B_1, N)$=0.7085
$\boldsymbol{d(B_2, P)}$=1.3735	$d(B_1, P)$=0.8645
$\boldsymbol{d(B_3, P)}$=1.4245	$d(B_1, P)$=1.3020
$\boldsymbol{d(B_4, P)}$=1.1615	$d(B_1, P)$=1.0765
$\boldsymbol{d(B_5, P)}$=0.9575	$d(B_1, P)$=1.2805

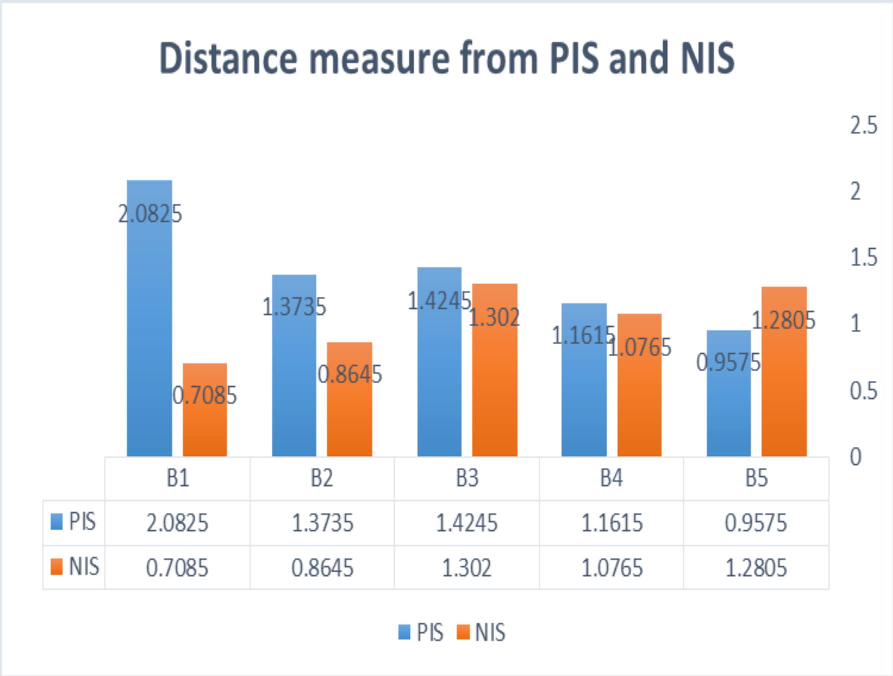


Figure 1: The distance measure of PIS and NIS

Finally we measure the relative closeness co-efficient of each alternative. The detail description have found in Table 12 and Figure 2.

Table 12: Relative Closeness Co-efficient

$C(B_1)$	$C(B_2)$	$C(B_3)$	$C(B_4)$	$C(B_5)$
0.746	0.613	0.522	0.518	0.427

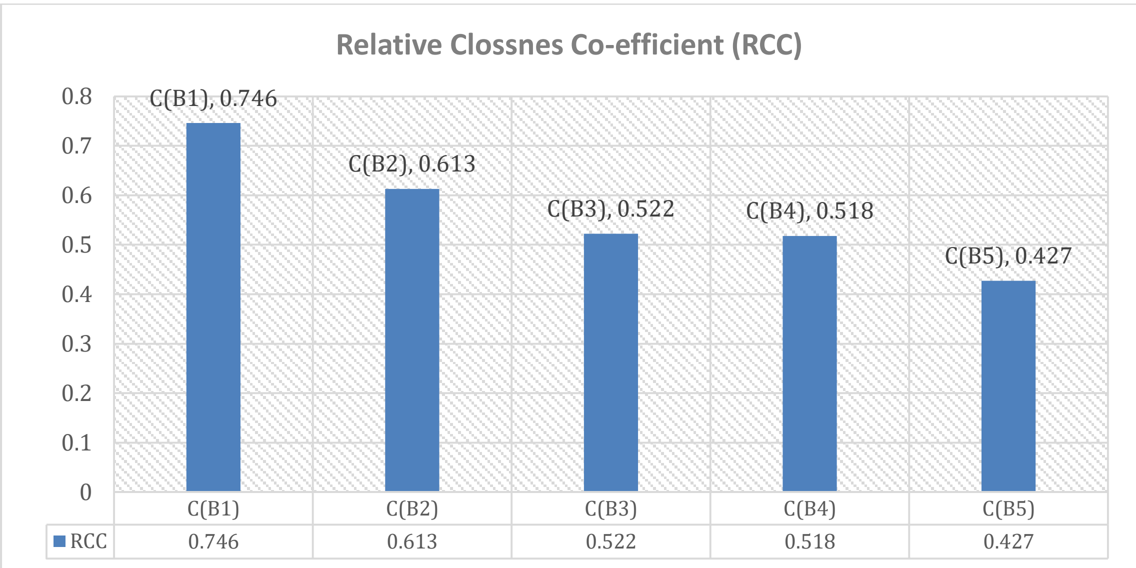


Figure 2: The relative closeness co-efficient

7. Conclusion

In this article, we started a series of bipolar fuzzy aggregation operators by using the idea of power aggregation operator. We obtained some different results related to bipolar fuzzy aggregation operator. We have applied our operators to develop some approaches to multiple attribute group decision making with bipolar fuzzy information. We developed an algorithm for a decision making problem based on bipolar fuzzy power aggregation. Further we extend our work to other type aggregation operators. We will develop power aggregation operators for bipolar fuzzy triangular numbers and will apply to selection of cluster head in smart grade and TOPSIS method.

Author contributions: All authors equally contributed to this manuscript.

Data Availability Statement: There is no associate data with this research.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] Zadeh, L.A., 1965. Fuzzy sets. *Information and control*, *8*(3), pp.338-353.

[2] Atanassov, K.T., 1999. Intuitionistic fuzzy sets. In *Intuitionistic fuzzy sets: theory and applications* (pp. 1-137). Heidelberg: Physica-Verlag HD.

[3] Ahmadzai, H.G. and Mohammadi, M.K., 2025. Development of Multi-Criteria Decision Models using COPRAS Method through Pythagorean Cubic Fuzzy Numbers. *Journal of Fuzzy Intelligence*, *1*(01), pp.1-9.

[4] Ahmad, S. and Zaman, S., 2025. A Novel Fuzzy Intelligence-Based Decision Support System and Its Application. *Journal of Fuzzy Intelligence*, *1*(01), pp.10-19.

[5] Almagharbeh, W.T., 2025. The impact of AI-based decision support systems on nursing workflows in critical care units. *International nursing review*, *72*(2), p.e13011.

[6] Levra, A.G., Gatti, M., Mene, R., Shiffer, D., Costantino, G., Solbiati, M., Furlan, R. and Dipaola, F., 2025. A large language model-based clinical decision support system for syncope recognition in the emergency department: A framework for clinical workflow integration. *European journal of internal medicine*, *131*, pp.113-120.

[7] S. M. Chen and J. M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets Syst., 67, 2, (1994), 163--172.

[8] V. Torra and Y. Narukawa, Modeling Decisions: Information Fusion and Aggregation Operators. Berlin, Germany: Springer-Verlag, 2007.

[9] W. Wang and X. Liu, Intuitionistic fuzzy information aggregation using Einstein operations, IEEE Trans. Fuzzy Syst., 20, 5, (2012), 923-938.

[10] Z. S. Xu, Intuitionistic fuzzy aggregation operators, IEEE Trans. Fuzzy Syst., 15, 6, (2007), 1179-1187.

[11] Z. S. Xu, Intuitionistic fuzzy information: Aggregation Theory and Applications. Beijing, China: Science, 2008.

[12] G. Wei, Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making, Appl. Soft. Comput., 10, 2, (2010), 423--431.

[13] D. F. Li, Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets, Expert Syst. Appl., 37, 12, (2010), 8673--8678.

[14] R. R. Yager, Generalized OWA aggregation operators, Fuzzy Optim. Decis. Mak., 3, 1, (2004), 93--107.

[15] Y. He, H. Chena, L. Zhoua, J. Liub and Z. Taoa, Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making, Inf. Sci., 259, (2014),142--159.

[16] S. Abdullah, N. Amin, Analysis of S-box image encryption based on generalized fuzzy soft expert set, 79, 3, (2015), 1679-1692.

[17] W.R. Zhang, Bipolar fuzzy sets and relations: a computational frame work for cognitive modelling and multiagent decision analysis, Proceedings of IEEE Conf., 1994, 305-309.

[18] W. R. Zhang, Bipolar fuzzy sets, Proceedings of FUZZY-IEEE, (1998), 835—840.

[19] I. Bloch, Bipolar fuzzy spatial information: geometry, morphology, spatial reasoning, Methods for Handling Imperfect Spatial Information, Springer Berlin Heidelberg, 256, 75-102.

[20] I. Bloch, Bipolar fuzzy mathematical morphology for spatial reasoning, Mathematical Morphology and Its Application to Signal and Image Processing, 5720 of the series Lecture Notes in Computer Science, 24-34.

[21] I. Bloch, Fuzzy and bipolar mathematical morphology, applications in spatial reasoning, Symbolic and Quantitative Approaches to Reasoning with Uncertainty, 5590 of the series Lecture Notes in Computer Science, 1-13.

[22] M. Akram, Bipolar fuzzy graphs, Inf. Sci., 181, 24, (2011), 5548--5564.

[23] H. Rashmanlou, S. Samanta, M. Pal and R.A. Borzooei, Bipolar fuzzy graphs with categorical properties, I. J. of Comput. Intell. Sys., 8, 5, (2015), 808-818.

[24] S. Samanta and M. Pal, Bipolar fuzzy hypergraphs, I. J. of Fuzzy Logic Sys., 2, 1, (2012), 17--28.

[25] S. Samanta and M. Pal, Irregular bipolar fuzzy graphs, I. J. App. Fuzzy Sets, 2, (2012), 91--102 .

[26] S. Samanta and M. Pal, Some more results on bipolar fuzzy sets and bipolar fuzzy intersection graphs, The J. Fuzzy Math., 22, 2, (2014), 253--262.

[27] P.K. Singh, C. A. Kumar, A note on bipolar fuzzy graph representation of concept lattice, 5,4, (2014), 381-393.