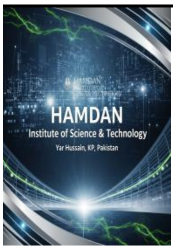

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Diagnosis of Depression Disorder Using Complex Non-Linear Diophantine Fuzzy Information

Maria Shams[1], Mubasir Hussain[2,*], Ikram Ullah[3]

[1]Department of Mathematics and Statistics, Women University Swabi, KP, Pakistan.

[2]*Department of Madicine, Medical A unit, Ayub Teaching Hospital, Abbottabad, KP, Pakistan.

[3]School of Mathematics and Statistics, Central South University, Changsha 410083, Hunan, P.R.China.

*Corresponding author: mubasir141@gmail.com

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ABSTRACT

Depression is recognized as a persistent mood illness. We develop a mathematical model of how clinically expert's psychiatrists identify signs of depression and provide a diagnosis. No matter what kind of therapy you select, psychotherapy should be a pleasant and supportive experience. You should always feel confident discussing your emotions and problems with depression when dealing with a psychotherapist. Five depression therapy techniques have been considered into a multi-criteria group decision-making (MCGDM) issue applying the Dombi geometric operator combining complex non-linear Diophantine fuzzy (CN-LDF) information. In real-world decision-making situations, most issues involve many factors that have to be considered. It may often be difficult to make decisions in these kinds of situations. We will need to use MCGDM procedures to prepare for such complex challenges. This study will provide a new approach called COmbinative Distance-Based Assessment (CODAS) and explore MCGDM problems. In this work, we developed new operators: CN-LDF Dombi weighted geometric (CN-LDFDWG), CN-LDF Dombi order weighted geometric (CN-LDFDOWG), and CN-LDF Dombi hybrid weighted geometric (CN-LDFDHWG). Further, the proposed model is implemented to solve a real-life problem related to the treatment therapy for depression disorder under CN-LDF Dombi Norms. The developed operators are more effective and reliable as compared to the existing operators.

Keywords

Complex non-linear Diophantine fuzzy Dombi weighted geometric (CN-LDFDWG) operators;
CODAS MCGDM problems;
Depression Diagnosis.

1. Introduction

Millions Of people throughout the world suffer from the disease of depression. Worldwide, depression affects more than 300 million people of all ages. The 21st century is marked by a depression epidemic. One in five people will experience a mood problem at some point in their lives. Ages 25 to 44 are when occurrences are most common. It is unclear and unknown what causes depression. According to theory, it happens as a result of the combination of biological factors, brain chemical imbalances, and personal problems. In the modern world, there are a lot of groups that do not consider depression to be a mental condition. Suicide can result from depression, the worst type of mental condition. What exactly is depression, is an often-asked question. "Depression is a frequent and serious psychiatric disorder that has a stressful effect on how you feel, think, and behave." A third of all cases of depression will be severe enough to need treatment from a doctor. The American Psychiatric Association claims that 80 to 90% of persons who are depressed can be healed. Getting a complete diagnosis from a medical or mental health expert is the first step. It is important to rule out any potential medical issues [1]. Psychotherapy and medicine are recommended by some treatment modalities. Others argue that treating symptoms with just medicine or treatment is sufficient. Each of these therapeutic strategies has some quantitative backing. Abdullah [2] proposed a different Electroencephalograms (EEG) classifiers based on five types of entropies under the LDFNs for analyzing patients suffering from depression.

1.1 The sequential way of predicting medical decisions

There are essentially two key stages in the medical decision-making process. The differential diagnostic (DD) is the first stage, and the provisional or final diagnosis (PD) is the second stage. The history of the patient and their symptoms are viewed as inputs by the physicians during differential diagnostic. This information is then used to diagnose diseases that overlap or appear to be related depending on their medical expertise and knowledge base. According to common perception, doctors may give symptoms arbitrary weights to determine how much of an impact they have on the total burden of an illness. Such weights are repeatedly examined and changed as necessary by numerous clinical examinations, or iterations in computer science. In clinical medicine, it is possible for different diseases to appear with equivalent patterns, and vice versa. In order to match the symptom patterns with the classical cases of each potential disease (obtained during differential diagnostic), doctors use the inputs as a starting point and then compare the similarities. They then rank the potential diseases and management techniques recommended with the top disease having the highest ranking based on the degree of similarity. All potential diseases (discovered through differential diagnostic) have their results from investigations and preliminary therapy closely matched, and the best-matching disease is then picked. Provisional diagnosis (PD) is the name given to this procedure. But in performance, the procedure is not as simple and straightforward. It takes multiple iterations to get from DD to PD [3].

1.2 Complications in diagnosing depression

The most complicated area of medical sciences is psychiatry. Due to their ambiguous clinical manifestations, psychiatric disorders are not directly determined. Manual connections between test and treatment results and morbidity progression have the potential for bias in decision. Depression frequently begins in a hidden way. It could result from a variety of non-psychiatric conditions, including cancers [4], persistent joint pain [5] and more. It is frequently noted that similar types of symptoms can occur with various diseases, and vice versa. Additionally, numerous disorders may be hiding under a particular set of symptoms, as is the case with depression and mania. As a result, clinical psychiatry's screening for depression (DD) and formal diagnosis (PD) processes are both lengthy and difficult.

Instead of creating a fully functional expert tool, the goal of this research is to mathematically describe the clinical decision-making process in psychiatry using soft computing. It is a cross-disciplinary effort that focuses solely on application-based research in mental health. Sadly, there isn't much of this type of interdisciplinary research, so this is an attempt to add to the body of' mental health-informatics' research that already exists. In the clinical research of depression, it is suggested that new methodologies be developed from scratch and then integrated in a model. To be viewed as the next step in the computerization of the diagnosis of depression, this is a future scope.

2. *Literature review*

Antidepressant medicines are efficient in treating depression, according to a number of research. As an illustration, one meta-analysis examined 300 previously published papers that looked at the treatment of depressive disorders [6]. The outcomes demonstrated that these antidepressants are efficient and that there aren't many quantitative differences between them. There may be variations in dosage or the kinds of side effects that are recorded. Numerous studies have been conducted to determine which of these two medications pharmacotherapy treatments (TCAs) or Selective Serotonin Reuptake Inhibitors (SSRIs) is most effective in the depression treatment. The Danish University Antidepressant Group (DUAG) published a study in 1986 that revealed conflicting results about the differences between the effectiveness of TCAs and Selective Serotonin Reuptake Inhibitors (SSRIs). The study comprised 114 inpatients with endogenous and non-endogenous depression who had received a diagnosis. Endogenous depression has melancholy characteristics associated with major depressive disorder [7]. Recent research has looked into the results of various drug combinations. To treat depression, for instance, [8] investigated the effects of combining norepinephrine and serotonin reuptake inhibition mechanisms. 38 inpatients with a diagnosis of nonpsychotic unipolar major depression were divided into several six-week therapies at random. There are several varieties of therapy available, and some of them appear to be effective in treating particular issues. Depression may be successfully treated with psychotherapy, according to research. As an illustration, [9] carried out a meta-analysis of 56 controlled studies with outcomes to demonstrate the efficiency of both pharmacological therapy and psychotherapy in the treatment of adult unipolar depression. We looked at behaviour treatment, interpersonal therapy, and a mix of cognitive, social learning, and behaviour therapies. According to the findings, psychotherapy has a substantially greater effect on unipolar depression. A mean size impact of 1.22 as opposed to 0.61, respectively, demonstrated that it was superior to medication treatment. Client preferences are a key benefit of psychotherapy over medical treatments [10,11]. The majority of patients favour psychotherapy, according to research on treatment preferences for depression. The causes of depression were thought to be resolved by psychotherapy. Due to the opportunity for patients to express their emotions and get support, more patients received the type of care that they preferred. Multi-criteria group decision making (MCGDM) is a method for resolving multiple alternative and conflicting criterion choice situations that is systematic and mathematical.

The theory of fuzzy set (FS) containing membership function (MF) were first presented by Zadeh [12] in 1965, is a useful tool for dealing with uncertain data in ordinary life. Thus, in 2002, [13] presented the theory of complex FS (CFS) and defined as a complex valued (CV). The CFS attracted a lot of interest within the context of FS. The MF and non-membership function (NMF) are specified by the generalization of FS known as intuitionistic FS (IFS) [14], with the limitation that the total of MF and NMF must be inside [0,1]. IFS has been applied by a number of researchers, containing [15,16,17,18,19,20,21,22,23,24]. In 2012, [25] proposed the concept of Complex IFS (CIFS), defined by the complex valued MF and NMF. Recently, [26,27] offered the CIF power aggregation operators (AOs) and many generalized CIF operators. Further [28] provided a strong correlation coefficient measure of CIFSs. Yager [29] introduced the Pythagorean fuzzy set (PyFS) with *(MF+NMF)²∈[0,1]*. Researcher [30] proposed an interaction between PyFNs and complex numbers. Regarding to complex PyF values, the concept of complex PyFS [31] was refined and extended over a wide range of distance measures. In 2017, Yager [32] developed q-rung orthopair fuzzy set (q-ROFS), provided that the qth power of MF and NMF bound between 0 and 1. The concept of q-ROFSs have been used by many academics in a range of fields, such as [33,34,35,36,37,38].

To build a decision framework, the author in [39] defines complex q-ROFS (Cq-ROFS). Due to the fact that reference parameters (RPs) cannot be supported by IFS, PyFS, or q-ROFS, Riaz [40] created the linear Diophantine fuzzy set (LDFS), by adopted the RPs concept. In 2021, [41] the idea of complex LDFSs (CLDFSs) were developed. Several researchers have used the concept of LDFSs, including [42,43,44,45,46]. Almagrabi et al. [47] developed and analyzed a unique LDF extension called the q-rung linear Diophantine fuzzy set (q-RLDFS) by adding the qth power to RPs. In 2021, the similarity measure for LDFNs were developed by [48]. In 2023, Shams et. al., [49] extended the q-RLDFS to non-linear Diophantine fuzzy set (N-LDFS) and applying it for complex numbers. Further the theory of CN-LDFS under Dombi Norms were defined by [50]. The Complex N-LDFS is more generalized than q-RLDFS and CLDFS because CN-LDFS presented the complex reference parameters (CV-RPs), which were presented for the first time.

2.1 Dombi operators for different MCGDM problems

Dombi [51] established the Dombi operations in 1982. To capitalize on these advantages, Liu et al. [52] created a multi-criteria group decision making (MCGDM) issue applying the Dombi Bonferroni mean operator for IF data. A MADM issue using Dombi operators were presented for a single-valued neutrosophic (SVN) environment by [53]. Based on the hesitant fuzzy data provided by the Dombi operators, he [54] shared his view on the Typhoon event. Dombi operations were utilized by Akram et al. [55] for a PyF sets. Currently most modified form of fuzzy are running to developed such as [56,57,58].

Contribution of work

The aim of this work is to developed the effective treatment or diagnosis for depression. The most effective course of treatment for a patient suffering from depression is determined using the MCGDM approach, which involves collecting a variety of expert opinions. We implemented the CODAS [59] method for CN-LDFNs based on Dombi Norms. The following contributions have been made by this research initiative in order to explore the MCGDM problem under ambiguity:

(1) A novel operation for CN-LDFNs is established to solve the real-life problems identified by Dombi operators. (2) Based on the Dombi operating rules, we provide three aggregation operators: CN-LDF Dombi weighted geometric (CN-LDFDWG), CN-LDFDOWG, and CN-LDFDHWG. (3) A model known as the CN-LDFD-CODAS approach is described in order to determine which diagnosis strategy is appropriate for a patient who is depressed. Although the suggested approach combines the expert opinion values utilizing the suggested Dombi operators, it is more efficient and successful to employ. (4) Using a case study of the depression diagnosis, we implement the recommended CN-LDFD-CODAS approach to show the applicability of our proposed method. We contextual the developed technique in order to assess the most effective depression treatment. (5) We contrast the proposed method with the previously employed techniques for the precision and acceptance of the proposed approach.

The structure of this work is as follows: Section 3 provides associated concepts to CN-LDFS. The operating rules of CN-LDFNs under Dombi operators and expectation score function (ESF) are explained in Section 4. The CN-LDFDWG, CN-LDFDOWG, and CN-LDFDHWG aggregation operators (AOs) are presented in Section 5, along with an analysis of some desired theorems. A new MCGDM problem for CN-LDFS under Dombi Norms is presented in Section 6 and is further broken down into two algorithms, one of which is the CODAS approach. In Section 7, a case study on depression diagnosis and a numerical example of solving two algorithms are covered. The proposed and existing methodologies are compared in Section 8. The conclusion and future direction is covered in Section 9.

3. Preliminaries

The essential concepts of complex non- linear Diophantine fuzzy set (CN-LDFS), q-rung linear Diophantine fuzzy set (q-RLDFS), and complex linear Diophantine fuzzy set (CLDFS) are covered here.

Definition 1: *[40] Consider M be a fixed set, then L as a LDFS is defined as:*

$$L = \{b, \langle G_L(b), H_L(b)\rangle, \langle \alpha, \beta\rangle) : b \in M\}$$

where $G_L(b), H_L(b), \alpha, \beta \in [0,1]$ *denoted the MF, NMF and RPs correspondingly with;*

$$0 \leq (\alpha)G_L(b) + (\beta)H_L(b) \leq 1 \forall b \in M,$$
$$\text{with } 0 \leq \alpha + \beta \leq 1.$$

Definition 2: *[41] Consider* M *be a fixed set, then* E *as a CLDFS is defined as:*

$$E = \{(b, \langle G_E(b), H_E(b)\rangle, \langle \alpha_E, \beta_E\rangle) : b \in M\}$$

and;

$$E = \{(b, \langle g_E(b)e^{i.2\pi\theta_{g_E}(b)}, h_E(b)e^{i.2\pi\xi_{h_E}(b)}\rangle, \langle \alpha_E, \beta_E\rangle) : b \in M\}$$

where $g_E(b)e^{i.2\pi\theta_{g_E}(b)}$ *and* $h_E(b)e^{i.2\pi\xi_{h_E}(b)}$ *denoted the complex valued (CV) MF and CV-NMF and RPs is denoted by* α_E, β_E *, i.e.* $g_E(b), h_E(b), \alpha_E, \beta_E, \theta_{g_E}(b), \xi_{h_E}(b) \in [0,1].$ *Where "i" (iota) is the imaginary part and its value is* $i = \sqrt{(-1)}$ *, where* $i^2 = -1$ *(henceforth), with*

$$0 \leq g_E(b)\alpha_E + h_E(b)\beta_L \leq 1 \text{ and } 0 \leq \theta_{g_E}(b)\alpha_E + \xi_{h_E}(b)\beta_L \leq 1.$$

Definition 3: *[47] Consider M be a fixed set, then S as a q-RLDFS is defined as:*

$$S = \{(b, \langle G_S(b), H_S(b)\rangle, \langle \alpha, \beta\rangle) : b \in M\}$$

where $G_S(b), H_S(b), \alpha, \beta \in [0,1]$ *denoted the MF, NMF and RPs correspondingly with;*

$$0 \leq (\alpha)^q G_S(b) + (\beta)^q H_S(b) \leq 1 \forall b \in M$$
$$\text{and } 0 \leq \alpha^q + \beta^q \leq 1,\ q \geq 1.$$

Shams [49] presented the idea of CN-LDFS along with complex valued RPs (CV-RPs) corresponding with their exponential. Additionally, Shams adopted the CN-LDF Dombi concept [50] from an averaging point of view. The theory of N-LDFS were then extended to Hamacher norms by [60]. Now, we expand the application of Dombi norms for CN-LDFNs to Dombi weighted geometric operators.

Definition 4: *[49] A complex non- linear Diophantine fuzzy set* A *on a fixed set* M *is defined as:*

$$A = \{(b, \langle G_A(b), H_A(b)\rangle, \langle \alpha_A, \beta_A\rangle): \ b \in M\} \qquad (1)$$

$$A = \{(b, \langle g_A(b)e^{i.2\pi\theta_{g_A}(b)}, h_A(b)e^{i.2\pi\xi_{h_A}(b)}\rangle, \langle \alpha_A e^{i.2\pi\phi_{\alpha_A}}, \beta_A e^{i.2\pi\eta_{\beta_A}}\rangle): \ b \in M\} \qquad (2)$$

where $g_A(b)$, $h_A(b)$, *are CV-MF and CV-NMF,* α_A *and* β_A *are CV-RPs,* $i = \sqrt{(-1)}$ *, where* $i^2 = -1$ *, (henceforth) and* $0 \leq \alpha_A^q + \beta_A^q \leq 1$ *, satisfying*

$$0 \leq g_A(b)\alpha_A^q + h_A(b)\beta_A^q \leq 1 \text{ and } 0 \leq \theta_{g_A}(b)\phi_{\alpha_A}^q + \xi_{h_A}(b)\eta_{\beta_A}^q \leq 1. \qquad (3)$$

Definition 5: [49] *Let* $U_j = \left\{ \begin{array}{l} \langle [g_j].e^{i2\pi[\theta_{g_j}]}, [h_j].e^{i2\pi[\xi_{h_j}]}\rangle, \\ \langle [\alpha_j].e^{i2\pi[\varphi_{\alpha_j}]}, [\beta_j].e^{i2\pi[\eta_{\beta_j}]}\rangle \end{array} \right\}$ *∈ CN-LDFNs, with* $q \geq 1, \quad K > 0.$ *Where CN-LDFNs are denoted by* U_j *and* U_j^c *denoted the complement of CN-LDFNs. Hence following are the fundamental operations of CN-LDFNs;*

$$(i)\ U_j^c = \left(\left\langle [h_j]\times e^{i2\pi\left[\xi_{h_j}\right]}, [g_j]\times e^{i2\pi\left[\theta_{g_j}\right]}\right\rangle, \left\langle [\beta_j]\times e^{i2\pi\left[\eta_{\beta_j}\right]}, [\alpha_j]\times e^{i2\pi\left[\varphi_{\alpha_j}\right]}\right\rangle\right)$$

$$(ii)\ U_1 \oplus U_2 = \left[\begin{array}{c} \left(\left[\sqrt[q]{g_1^q + g_2^q - g_1^q g_2^q}\right]\times e^{i2\pi\left[\sqrt[q]{\theta_{g_1}^q+\theta_{g_2}^q-\theta_{g_1}^q\theta_{g_2}^q}\right]}, [h_1 h_2]\times e^{i2\pi\left[\xi_{h_1}\xi_{h_2}\right]}\right), \\ \left(\left[\sqrt[q]{\alpha_1^q + \alpha_2^q - \alpha_1^q \alpha_2^q}\right]\times e^{i2\pi\left[\sqrt[q]{\varphi_{\alpha_1}^q+\varphi_{\alpha_2}^q-\varphi_{\alpha_1}^q\varphi_{\alpha_2}^q}\right]}, [\beta_1 \beta_2]\times e^{i2\pi\left[\eta_{\beta_1}\eta_{\beta_2}\right]}\right) \end{array}\right]$$

$$(iii)\ U_1 \otimes U_2 = \left[\begin{array}{c} \left([g_1 g_2]\times e^{i2\pi\left[\theta_{g_1}\theta_{g_2}\right]}, \left[\sqrt[q]{h_1^q + h_2^q - h_1^q h_2^q}\right]\times e^{i2\pi\left[\sqrt[q]{\xi_{h_1}^q+\xi_{h_2}^q-\xi_{h_1}^q\xi_{h_2}^q}\right]}\right), \\ \left([\alpha_1 \alpha_2]\times e^{i2\pi\left[\varphi_{\alpha_1}\varphi_{\alpha_2}\right]}, \left[\sqrt[q]{\beta_1^q + \beta_2^q - \beta_1^q \beta_2^q}\right]\times e^{i2\pi\left[\sqrt[q]{\eta_{\beta_1}^q+\eta_{\beta_2}^q-\eta_{\beta_1}^q\eta_{\beta_2}^q}\right]}\right) \end{array}\right]$$

$$(iv)\ k.U = \left[\begin{array}{c} \left(\left[\sqrt[q]{\left(1-(1-g^q)^k\right)}\right]\times e^{i2\pi\left[\sqrt[q]{\left(1-\left(1-\theta_g^q\right)^k\right)}\right]}, [h^k]\times e^{i2\pi\left[\xi_h^k\right]}\right), \\ \left(\left[\sqrt[q]{\left(1-(1-\alpha^q)^k\right)}\right]\times e^{i2\pi\left[\sqrt[q]{\left(1-\left(1-\varphi_\alpha^q\right)^k\right)}\right]}, [\beta^k]\times e^{i2\pi\left[\eta_\beta^k\right]}\right) \end{array}\right]$$

$$(v)\ U^K = \left[\begin{array}{c} \left([g^k]\times e^{i2\pi\left[\theta_g^k\right]}, \left[\sqrt[q]{\left(1-(1-h^q)^k\right)}\right]\times e^{i2\pi\left[\sqrt[q]{\left(1-\left(1-\xi_h^q\right)^k\right)}\right]}\right), \\ \left([\alpha^k]\times e^{i2\pi\left[\varphi_\alpha^k\right]}, \left[\sqrt[q]{\left(1-(1-\beta^q)^k\right)}\right]\times e^{i2\pi\left[\sqrt[q]{\left(1-\left(1-\eta_\beta^q\right)^k\right)}\right]}\right) \end{array}\right]$$

Now we will define the Dombi operations for complex non- linear Diophantine fuzzy numbers (CN-LDFNs) and further their important properties.

4. Operational Laws of CN-LDF Dombi Norms

Here we presented the operational laws of CN-LDFNs on Dombi.

Definition 6: *[50] Suppose* $U_j = \left\{\begin{array}{c} \left\langle [g_j].e^{i2\pi\left[\theta_{g_j}\right]}, [h_j].e^{i2\pi\left[\xi_{h_j}\right]}\right\rangle, \\ \left\langle [\alpha_j].e^{i2\pi\left[\varphi_{\alpha_j}\right]}, [\beta_j].e^{i2\pi\left[\eta_{\beta_j}\right]}\right\rangle \end{array}\right\}$ *belong to CN-LDFNs. Then, the basic operations of Dombi for CN-LDFNs with* $\Psi, q, K \geq 1$ *is described as follow;*

$$(1)U_1 \oplus U_2 = \left[\begin{gathered}\left(\begin{gathered}\left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{g_1^q}{1-g_1^q}\right)^{\Psi}+\left(\frac{g_2^q}{1-g_2^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right].e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{\theta_{g_1}^q}{1-\theta_{g_1}^q}\right)^{\Psi}+\left(\frac{\theta_{g_2}^q}{1-\theta_{g_2}^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]}, \\ \left[\frac{1}{1+\left\langle\left(\frac{1-h_1}{h_1}\right)^{\Psi}+\left(\frac{1-h_2}{h_2}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right].e^{i2\pi\left[\frac{1}{1+\left\langle\left(\frac{1-\xi_{h_1}}{\xi_{h_1}}\right)^{\Psi}+\left(\frac{1-\xi_{h_2}}{\xi_{h_2}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}\end{gathered}\right), \\ \left(\begin{gathered}\left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{\alpha_1^q}{1-\alpha_1^q}\right)^{\Psi}+\left(\frac{\alpha_2^q}{1-\alpha_2^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right].e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{\phi_{\alpha_1}^q}{1-\phi_{\alpha_1}^q}\right)^{\Psi}+\left(\frac{\phi_{\alpha_2}^q}{1-\phi_{\alpha_2}^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]}, \\ \left[\frac{1}{1+\left\langle\left(\frac{1-\beta_1}{\beta_1}\right)^{\Psi}+\left(\frac{1-\beta_2}{\beta_2}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right].e^{i2\pi\left[\frac{1}{1+\left\langle\left(\frac{1-\eta_{\beta_1}}{\eta_{\beta_1}}\right)^{\Psi}+\left(\frac{1-\eta_{\beta_1}}{\eta_{\beta_1}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}\end{gathered}\right)\end{gathered}\right] \quad (4)$$

$$(2) U_1 \otimes U_2 = \left[\begin{array}{l} \left(\begin{array}{l} \left[\frac{1}{1+\left\langle\left(\frac{1-g_1}{g_1}\right)^{\Psi}+\left(\frac{1-g_2}{g_2}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right] . e^{i2\pi\left[\frac{1}{1+\left\langle\left(\frac{1-\theta_{g_1}}{\theta_{g_1}}\right)^{\Psi}+\left(\frac{1-\theta_{g_2}}{\theta_{g_2}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}, \\ \left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{h_1^q}{1-h_1^q}\right)^{\Psi}+\left(\frac{h_2^q}{1-h_2^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right] . e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{\xi_{h_1}^q}{1-\xi_{h_1}^q}\right)^{\Psi}+\left(\frac{\xi_{h_2}^q}{1-\xi_{h_2}^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]} \end{array}\right), \\ \left(\begin{array}{l} \left[\frac{1}{1+\left\langle\left(\frac{1-\alpha_1}{\alpha_1}\right)^{\Psi}+\left(\frac{1-\alpha_2}{\alpha_2}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right] . e^{i2\pi\left[\frac{1}{1+\left\langle\left(\frac{1-\phi_{\alpha_1}}{\phi_{\alpha_1}}\right)^{\Psi}+\left(\frac{1-\phi_{\alpha_1}}{\phi_{\alpha_1}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}, \\ \left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{\beta_1^q}{1-\beta_1^q}\right)^{\Psi}+\left(\frac{\beta_2^q}{1-\beta_2^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right] . e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\left(\frac{\eta_{\beta_1}^q}{1-\eta_{\beta_1}^q}\right)^{\Psi}+\left(\frac{\eta_{\beta_2}^q}{1-\eta_{\beta_2}^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]} \end{array}\right) \end{array}\right] \quad (5)$$

$$(3)k.U = \left[\begin{matrix}\left(\begin{matrix}\left[\sqrt[q]{1-\frac{1}{1+\left\langle k\left(\frac{g^q}{1-g^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right].e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle k\left(\frac{\theta_g^q}{1-\theta_g^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]},\\ \left[\frac{1}{1+\left\langle k\left(\frac{1-h}{h}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right].e^{i2\pi\left[\frac{1}{1+\left\langle k\left(\frac{1-\xi_h}{\xi_h}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}\end{matrix}\right),\\ \left(\begin{matrix}\left[\sqrt[q]{1-\frac{1}{1+\left\langle k\left(\frac{\alpha^q}{1-\alpha^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right].e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle k\left(\frac{\phi_\alpha^q}{1-\phi_\alpha^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]},\\ \left[\frac{1}{1+\left\langle k\left(\frac{1-\beta}{\beta}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right].e^{i2\pi\left[\frac{1}{1+\left\langle k\left(\frac{1-\eta_\beta}{\eta_\beta}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}\end{matrix}\right)\end{matrix}\right] \quad (6)$$

$$(4)U^K = \left[\begin{matrix}\left(\begin{matrix}\left[\sqrt[q]{1-\frac{1}{1+\left\langle K\left(\frac{g^q}{1-g^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right].e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle K\left(\frac{\theta_g^q}{1-\theta_g^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]},\\ \left[\frac{1}{1+\left\langle K\left(\frac{1-h}{h}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right].e^{i2\pi\left[\frac{1}{1+\left\langle K\left(\frac{1-\xi_h}{\xi_h}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}\end{matrix}\right),\\ \left(\begin{matrix}\left[\sqrt[q]{1-\frac{1}{1+\left\langle K\left(\frac{\alpha^q}{1-\alpha^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right].e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle K\left(\frac{\phi_\alpha^q}{1-\phi_\alpha^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]},\\ \left[\frac{1}{1+\left\langle K\left(\frac{1-\beta}{\beta}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right].e^{i2\pi\left[\frac{1}{1+\left\langle K\left(\frac{1-\eta_\beta}{\eta_\beta}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}\end{matrix}\right)\end{matrix}\right] \quad (7)$$

Definition 7: *[49] Suppose T be a fixed set and* $U \in CN-LDFNs$, *then expectation score function (ESf) of U is defined as;*

$$ES(U) = \left[\frac{g(\theta_g) - h(\xi_h) + 1}{4} + \frac{\alpha^q(\phi_\alpha^q) - \beta^q(\eta_\beta^q) + 1}{4}\right]; q \geq 1 \qquad (8)$$

The ESF bound to $[0,1]$.

(i) If $ES_f(U_1) < ES_f(U_2)$, then $U_1 < U_2$

(ii) If $ES_f(U_1) > ES_f(U_2)$, then $U_1 > U_2$

(iii) If $ES_f(U_1) = ES_f(U_2)$, then $U_1 = U_2$.

5. *Aggregation Operator (AOs)*

We have proposed the AOs for CN-LDF data under Dombi Norms throughout this area. Section 5.1 presented the CN-LDFDWG, CN-LDFDOWG and CN-LDFDHWG AOs.

5.1 Complex N-LDF Dombi Weighted Geometric AOs

We developed the Dombi AOs for CN-LDF from the geometric aspect namely CN-LDFDWG, CN-LDFDOWG and CN-LDFDHWG, along with their definitions and basic theorems.

Theorem 1: *Suppose* $U_j = \left\{ \begin{array}{l} \left\langle [g_j] \times e^{i2\pi[\theta_{g_j}]}, [h_j] \times e^{i2\pi[\xi_{h_j}]} \right\rangle, \\ \left\langle [\alpha_j] \times e^{i2\pi[\varphi_{\alpha_j}]}, [\beta_j] \times e^{i2\pi[\eta_{\beta_j}]} \right\rangle \end{array} \right\} : j \in \mathbb{N}$ *be a collection of CN-LDFNs on an arbitrary set T and* $£ = (£_1, £_2, \ldots, £_n)^T$ *identified the weights i.e.* $\sum_{j=1}^{n} £_j = 1,$ *thus, by implementing the CN-LDFDWG AOs their (combined value is again a CN-LDFNs, the transformation on* χ *is called CN-LDF Dombi weighted geometric (CN-LDFDWG) AOs and described as;*

$$CN - LDFDWG(U_1, \ldots, U_n) = \bigotimes_{j=1}^{n} U_j^{£_j}$$

$$
=\left[\begin{array}{c}
\left(\begin{array}{c}
\left[\dfrac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-g_j}{g_j}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]\times e^{i2\pi\left[\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-\theta_{gj}}{\theta_{gj}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]},\\
\left[\sqrt[q]{1-\dfrac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{h_j^q}{1-h_j^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]\times e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{\xi_{hj}^q}{1-\xi_{hj}^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]}
\end{array}\right),\\
\left(\begin{array}{c}
\left[\dfrac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-\alpha_j}{\alpha_j}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]\times e^{i2\pi\left[\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-\varphi_{\alpha j}}{\varphi_{\alpha j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]},\\
\left[\sqrt[q]{1-\dfrac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{\beta_j^q}{1-\beta_j^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]\times e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{\eta_{\beta j}^q}{1-\eta_{\beta j}^q}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]}
\end{array}\right)
\end{array}\right] \tag{9}
$$

Following are the Idempotency, Boundedness and Monotonicity theorems respectively for CN-LDFNs.

1). If $U_j (j \in \mathbb{N})$ identified the collection of CN-LDFNs, are all identical i.e. $U_j = U\ \forall j,$ then

$$CN-LDFDWG(U_1,\dots,U_n)=U \qquad (10)$$

2). If $U_j (j \in \mathbb{N})$ combines a collection of CN-LDFNs, and suppose $U^- = \min_j U_j,\ U^+ = \max_j U_j,$ then

$$U^- \le CN-LDFDWG_£(U_1,\dots,U_n) \le U^+ \qquad (11)$$

3). Consider U_j and $U_j^* \ (j \in \mathbb{N})$ identified the two collection of CN-LDFNs, if $U_j \le U_j^*, \forall\ j,$ then

$$CN-LDFDWG_£(U_1,\dots,U_n) \le CN-LDFDWG_£(U_1^*,\dots,U_n^*) \qquad (12)$$

Definition 9: *Suppose* $U_j (j \in \mathbb{N})$ *identified the family of CN-LDFNs on an arbitrary set* T *and* $£ = (£_1, £_2, \dots, £_n)^T$ *identified the weights i.e.* $\sum_{j=1}^{n} £_j = 1,$ *then CN-LDF Dombi ordered weighted geometric (CN-*

LDFDOWG) AO is defined on the mapping χ *as;*

$$CN-LDFDOWG(U_1,\ldots,U_n) = \bigotimes_{j=1}^{n}(U_{(\delta)j})^{£_j} = U_{(\delta)1}^{£_1}\otimes\ldots\otimes U_{(\delta)n}^{£_n}. \tag{13}$$

The permutation of $(j \in \mathbb{N})$ *is* $(\delta(1),\ldots,\ \delta(n))$ *, for* $U_{\delta(j=1)} \geq U_{\delta(j)}\forall j.$

Theorem 2: *Suppose* $U_j = \left\{ \begin{array}{c} \left\langle [g_j].e^{i2\pi[\theta_{g_j}]},[h_j].e^{i2\pi[\xi_{h_j}]} \right\rangle, \\ \left\langle [\alpha_j].e^{i2\pi[\varphi_{\alpha_j}]},[\beta_j].e^{i2\pi[\eta_{\beta_j}]} \right\rangle \end{array} \right\} : j \in \mathbb{N}$ *be the family of CN-LDFNs on an arbitrary set T and* $£ = (£_1,£_2,\ldots,£_n)^T$ *listed the weights i.e.* $\sum_{j=1}^{n}£_j = 1,$ *and* $q,\Psi \geq 1;$ *thus by implementing the CN-LDFDOWG AOs their combined value is also a CN-LDFNs, their transformation on* χ *is known as CN-LDFDOWG AO and defined as;*

$$CN-LDFDOWG(U_1,\ldots,U_n) = \bigotimes_{j=1}^{n}(U_{(\delta)j})^{£_j}$$

$$= \left[\begin{array}{c}\left(\left[\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-g_{(\delta)j}}{g_{(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]\times e^{i2\pi\left[\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-\theta_{g(\delta)j}}{\theta_{g(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}, \left[\sqrt[q]{1-\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{h^q_{(\delta)j}}{1-h^q_{(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]\times e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{\xi^q_{h(\delta)j}}{1-\xi^q_{h(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]}\right), \\ \left(\left[\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-\alpha_{(\delta)j}}{\alpha_{(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]\times e^{i2\pi\left[\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{1-\varphi_{\alpha(\delta)j}}{\varphi_{\alpha(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}\right]}, \left[\sqrt[q]{1-\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{\beta^q_{(\delta)j}}{1-\beta^q_{(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]\times e^{i2\pi\left[\sqrt[q]{1-\frac{1}{1+\left\langle\prod_{j=1}^{n}£_j\left(\frac{\eta^q_{\beta(\delta)j}}{1-\eta^q_{\beta(\delta)j}}\right)^{\Psi}\right\rangle^{\frac{1}{\Psi}}}}\right]}\right)\end{array}\right] \quad (14)$$

The permutation/order of $(j \in \mathbb{N})$ *is* $(\delta(1), \delta(2), \ldots, \delta(n))$*, for* $U_{\delta(j=1)} \geq U_{\delta(j)} \forall j$.

Next, we presented the CN-LDFDHWG aggregation operator.

Definition 10: *Suppose* $U_j (j \in \mathbb{N})$ *identified the family of CN-LDFNs on an arbitrary set* T *and* $£ = (£_1, £_2, \ldots, £_n)^T$ *listed the weights i.e.* $\sum_{j=1}^{n} £_j = 1,$ *and* $q, \Psi \geq 1;$ *then CN-LDF Dombi hybrid weighted geometric (CN-LDFDHWG) AO is presented as;*

$$CN-LDFDHWG(U_1, \ldots, U_n) = \bigotimes_{j=1}^{n} (U^{\star}_{(\delta)j})^{£_j} = (U^{\star}_{(\delta)1})^{£_1} \otimes \ldots \otimes (U^{\star}_{(\delta)n})^{£_n}. \quad (15)$$

Theorem 3: *Suppose* $U_j = \left\{ \begin{array}{l} \left\langle [g_j].e^{i2\pi[\theta_{g_j}]}, [h_j].e^{i2\pi[\xi_{h_j}]} \right\rangle, \\ \left\langle [\alpha_j].e^{i2\pi[\varphi_{\alpha_j}]}, [\beta_j].e^{i2\pi[\eta_{\beta_j}]} \right\rangle \end{array} \right\} : j \in \mathbb{N}$ *identified the family of CN-LDFNs on an arbitrary set* T *and* $£ = (£_1, £_2, \ldots, £_n)^T$ *listed the weights i.e.* $\sum_{j=1}^{n} £_j = 1,$ *thus, by implementing the CN-LDFDHWG AOs their combined value is also a CN-LDFNs, their mapping on* χ *is called the CN-LDFDHWG AO and presented as;*

$$CN-LDFDHWG(U_1, \ldots, U_n) = \bigotimes_{j=1}^{n} (U^{\star}_{(\delta)j})^{£_j}$$

$$= \left[\begin{array}{c} \left(\begin{array}{c} \left[\dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{1-g^{\star}_{(\delta)j}}{g^{\star}_{(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}} \right] \times e^{i2\pi \left[\dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{1-\theta^{\star}_{g(\delta)j}}{\theta^{\star}_{g(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}} \right]}, \\ \left[\sqrt[q]{1 - \dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{h^{\star q}_{(\delta)j}}{1-h^{\star q}_{(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}} \right] \times e^{i2\pi \left[\sqrt[q]{1 - \dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{\xi^{\star q}_{h(\delta)j}}{1-\xi^{\star q}_{h(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}} \right]} \end{array} \right), \\ \left(\begin{array}{c} \left[\dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{1-\alpha^{\star}_{(\delta)j}}{\alpha^{\star}_{(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}} \right] \times e^{i2\pi \left[\dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{1-\varphi^{\star}_{\alpha(\delta)j}}{\varphi^{\star}_{\alpha(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}} \right]}, \\ \left[\sqrt[q]{1 - \dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{\beta^{\star q}_{(\delta)j}}{1-\beta^{\star q}_{(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}} \right] \times e^{i2\pi \left[\sqrt[q]{1 - \dfrac{1}{1+\left\langle \prod_{j=1}^{n} £_j \left(\frac{\eta^{\star q}_{\beta(\delta)j}}{1-\eta^{\star q}_{\beta(\delta)j}} \right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}} \right]} \end{array} \right) \end{array} \right] \quad (16)$$

Where $U^{\star}_{\delta(j)}$ *is the* jth *largest CN-LDF Dombi weighted values* $U^{\star}_i (U^{\star}_j = (U_j)^{n£_j}, j \in \mathbb{N}).$

Thus, we analyzed that CN-LDFDHWG operator is the extended version of CN-LDFDWG and CN-LDFDOWG AOs.

6. MCGDM model for CN-LDF Dombi geometric operator

Two MCGDM algorithms for complex N-LDFDW geometric AOs are developed and presented in this section. In the beginning, Section 6.1 discusses the formulation of the CN-LDFDWG strategy decision matrix. Next, Section 6.2 covers an explanation of the methods involved in developing the CN-LDF Dombi operators. Lastly, Section 6.3 outlines the procedures for CN-LDFDWG operators to implement the CODAS technique. The graphically steps for the suggested techniques are shown in Fig. 1.

6.1 Formulation of MCGDM Models

Here we have introduced two methods for CN-LDFDWG operators. This approach is being presented with the intention of assessing the viability and adaptability of the CN-LDFN and Dombi AOs. Consider $₣ = \{₣_1, ₣_2, \ldots, ₣_m\}$ identified the set of alternatives and $H = \{H_1, H_2, \ldots H_n\}$ identified the criteria along with weights $£ = (£_1, £_2, \ldots, £_n)^T$ i.e. $£_j > 0$, $\sum_{j=1}^{n} £_j = 1.$ And decision matrix (DMs) $DM_k(k = 1,2,\ldots,e)$ is considered by e, along with alternatives $₣_i$ and H_j criteria by using CN-LDFDWG operators. Equation (17) represented the kth DM:

$$\left[U_{ij}^{(k)}\right]_{m\times n} = \begin{bmatrix} \left\langle \left[g_{U_{ij}}^{(k)}\right] \times e^{i2\pi\left[\theta_{gU_{ij}}^{(k)}\right]}, \left[h_{U_{ij}}^{(k)}\right] \times e^{i2\pi\left[\xi_{hU_{ij}}^{(k)}\right]} \right\rangle, \\ \left\langle \left[\alpha_{U_{ij}}^{(k)}\right] \times e^{i2\pi\left[\phi_{\alpha U_{ij}}^{(k)}\right]}, \left[\beta_{U_{ij}}^{(k)}\right] \times e^{i2\pi\left[\eta_{\beta U_{ij}}^{(k)}\right]} \right\rangle \end{bmatrix}_{m\times n} \tag{17}$$

Two algorithms were designed for CN-LDFDWG operators. Following are the decision-making steps.

6.2 Algorithm-1: Complex N-LDFD Method

Step-1: Develop the DMs tables in CN-LDFN format for each possible alternative and criteria. $DM = \{DM_1, DM_2, \ldots, DM_k\}$ is identified the DM group.

Step-2: Because of several types of criteria, the CN-LDF data will be normalized.

$$N_{ij}^{(k)} = \begin{cases} U_{ij}^{(k)} & fob \quad \text{Benefit criteria} \\ (U_{ij}^{(k)})^c & fob \quad \text{Cost criteria} \end{cases} \tag{18}$$

Step-3: Assume the weights associated with each DM is, $£ = (£_1, £_2, £_3)^T$.

Step-4: Utilizing the CN-LDFDWG operators, calculate the $DM_k(k = 1,2,3)$ matrix into combined CN-LDF with weights $£ = (£_1, £_2, £_3)^T$ of criteria H_j.

Step-5: The result obtained in step-4 is computed again in a unified matrix with weights $£ = (£_1, £_2, £_3, £_4)^T$ for each criterion H_j , applying equation 9, 14 and 16 of CN-LDFDWG AOs.

Step-6: Applying equation. 8, to calculate the ESf of $₣_i$ (alternatives) from the combined table.

Step-7: Sort the alternatives according to which one has a higher ESf value.

6.3 Algorithm-2: Complex N-LDFD CODAS Model

To address MCGDM issues, [59] proposed the combinative distance-based assessment (CODAS) technique. The CODAS method combines the taxicab and Euclidean distances. These distances calculated the negative-ideal point, having greater distance of alternative is the best one [61].

Step-1: Develop the DMs table in CN-LDFN format for each possible alternative and criteria. $DM = \{DM_1, \ldots, DM_k\}$ is identified the DM group. The kth DM is presented in above equation 17.

Step-2: Compute the normalized decision matrix (N_{ij}), by the means of outlined below equation 19.

$$N_{ij} = \begin{cases} \dfrac{U_{ij}}{\max_i U_{ij}} & when \quad J \in Benifit \\ \dfrac{\min_i U_{ij}}{U_{ij}} & when \quad J \in Cost \end{cases} \tag{19}$$

Step-3: The weighted normalized performance values are computed by applying the formula as follow:

$$b_{ij} = \pounds_j \otimes N_{ij} = \left[\begin{array}{c} \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{g_j^q}{1-g_j^q}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}}\right] . e^{i2\pi \left[\sqrt[q]{1 - \frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{\theta_{gj}^q}{1-\theta_{gj}^q}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}}\right]}, \\ \left[\frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{1-h_j}{h_j}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}\right] . e^{i2\pi \left[\frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{1-\xi_{hj}}{\xi_{hj}}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}\right]} \end{array}\right), \\ \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{\alpha_j^q}{1-\alpha_j^q}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}}\right] . e^{i2\pi \left[\sqrt[q]{1 - \frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{\varphi_{\alpha j}^q}{1-\varphi_{\alpha j}^q}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}}\right]}, \\ \left[\frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{1-\beta_j}{\beta_j}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}\right] . e^{i2\pi \left[\frac{1}{1+\left\langle \prod_{j=1}^{n} \pounds_j \left(\frac{1-\eta_{\beta j}}{\eta_{\beta j}}\right)^{\Psi} \right\rangle^{\frac{1}{\Psi}}}\right]} \end{array}\right) \end{array}\right] \quad (20)$$

where $\sum_{j=1}^{n} \pounds_j = 1$ and $\pounds_j$ specifies the jth criterion weight.

Step-4: The process for identifying the negative-ideal solution (point (NS_j)) is presented below:

$$NS = [NS_j]_{1\times n} \quad (21)$$

$$NS_j = \min_i b_{ij} \quad (22)$$

Step-5: The Euclidean (E_i) and Taxicab (T_i) distances for the alternatives from the NS_j, is computed as follow.

$$E_i = \sqrt{\sum_{j=1}^{n} d_E (b_{ij} - N_j)^2} \quad (23)$$

$$T_i = \sum_{j=1}^{n} d_T |b_{ij} - N_j| \quad (24)$$

Step-6: Construct the relative assessment matrix, appears as:

$$bl = [p_{ik}]_{m \times m} \quad (25)$$

$$p_{ik} = (E_i - E_k) + \left(\Psi(E_i - E_k) \times (T_i - T_k)\right), \quad (26)$$

where Ψ is a threshold function that may be used to calculate the E_i and T_i distances between two alternatives that are equivalent, and k is a value within the range of the function $(k \in \mathbb{N})$.

$$\Psi(x) = \begin{cases} 1 & if \ |x| \geq \tau \\ 0 & if \ |x| \leq \tau \end{cases} \quad (27)$$

The function "τ" allows the decision-maker to set the Ψ parameter. The recommended range for this parameter's value is bound between $[0.01, 0.05]$. The T_i distance is used to compare the two alternatives if the difference between their E_i distances is less than τ. We assumed $\tau = 0.02$ in this calculation.

Step-7: Evaluate the assessment score (AS_i) of each alternative by using the formula as:

$$AS_i = \sum_{k=1}^{m} p_{ik} \quad (28)$$

Step-8: The ranking of alternatives are based on the decreasing values of AS_i. An alternative with the greatest AS_i is the best one among the choices. Following Figure. 1 shows graphically Algorithm of proposed method.

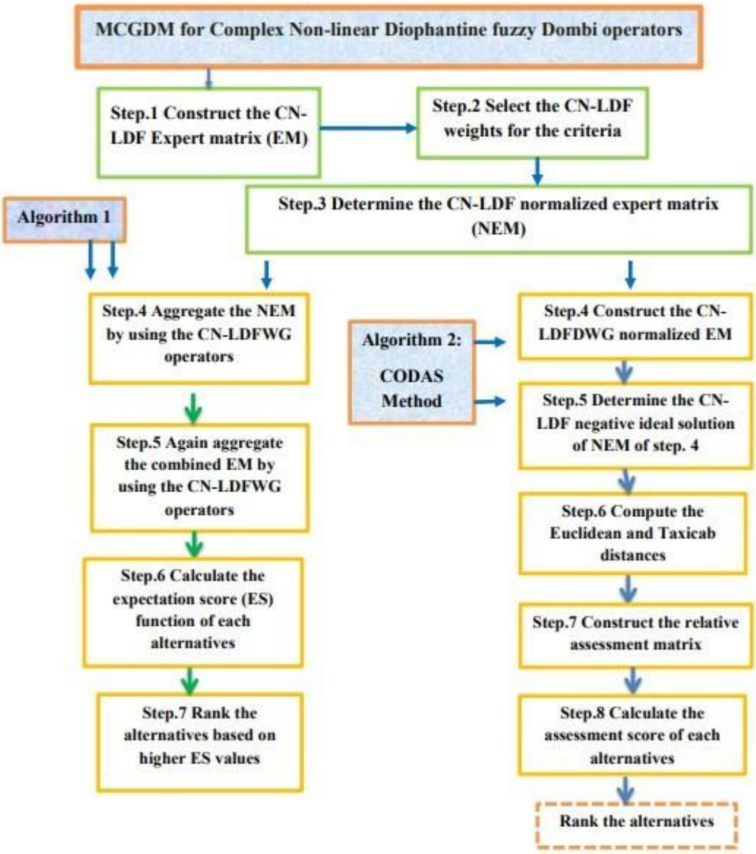

Figure 1 Developed CN-LDFD Algorithms

7. NUMERICAL EXAMPLE

For CN-LDF information based on Dombi Norms which includes CN-LDFDWG, CN-LDFDOWG and CN-LDFDHWG AOs, we developed a MCGDM issue. In Section 7.1, a real-world medical case study is presented to illustrate the feasibility of the suggested operators for diagnosing depression and medical treatment. The steps and output of algorithm-1 are provided in Section 7.2. The CODAS technique (algorithm-2) is presented in Section 7.3 for solving the numerical example.

7.1 Case Study: DIAGNOSIS OF DEPRESSION

A serious illness called depression has a damaging effect on a person's mental state, emotion, and behavior. In contrast to natural sorrow, clinical depression is long-term, frequently damages the capacity to enjoy or anticipate pleasure, and considerably discourages day-to-day functioning. The symptoms of depression have been present for at least two weeks to be identified. A medical evaluation at the office of a doctor is the initial step towards receiving a diagnosis. The symptoms of thyroid disorders and several medical diseases, including some prescription drugs, might be similar to those of depression. People experienced some forms of depression very badly, which contributed to the problem's criteria.

$(\mathbf{H}_1)$ **Persistent depressive disorder (PDD):** Mild to moderate depression that lasts for at least two years is referred to as PDD. Compared to major depressive illnesses, the symptoms are less serious. PDD used to be known as dysthymia by medical professionals.

$(\mathbf{H}_2)$ **Disruptive mood dysregulation disorder (DMDD):** In children with DMDD, there is a long-lasting, sharp irritation and a lot of uncontrolled anger. By the age of 10, symptoms often appear.

$(\mathbf{H}_3)$ **Premenstrual dysphoric disorder (PMDD):** Premenstrual syndrome (PMS) symptoms and mood symptoms, such as dangerous irritability, anxiety, or sadness, occur in PMDD.

$(\mathbf{H}_4)$ **Clinical depression (major depressive disorder):** You must experience these feelings on a daily basis for at least two weeks in addition to other symptoms such as difficulty sleeping, a lack of interest in previously enjoyed activities, or a change in hunger to receive a diagnosis of major depressive illness. This is one of the most prevalent and serious types of depression. A physical examination, an interview, and laboratory testing can be used by a clinician to rule out these possibilities. When an illness is ruled out as the cause, the doctor has two options: treat the patient directly or send them to a mental health specialist. A person with depression can be treated in a number of ways once they have been identified. Psychotherapy, which can also be used in combination with antidepressant medicines, is the cornerstone of treatment for depression. Researchers are attempting to better understand, identify, and treat depression in all population of groups. The essential objective is to research different alternatives to personalized care for depression, such as figuring out specific traits that suggest which therapies will be more effective. The techniques for treating the depression symptoms outlined above can be implemented to diagnose a patient who is depressed.

(1) Medications (F_1)**:** Medications called antidepressants are often used to treat depression. They often take 4 to 8 weeks to start working, and before mood gets higher, symptoms like sleep, hunger, or attention issues frequently get improved. Before concluding if the medication is effective or not, it is essential to give it a chance. When using antidepressants, especially in the first few weeks or after a dose changes, some people, especially children, teens, and young adults, may suffer an increase in suicidal thoughts or behavior. When a patient tries at least two antidepressants and still doesn't feel better, they may have treatment-resistant depression.

(2) Parent-child therapy (F_2)**:** A clinical research report suggests that young children with depression can benefit from an interactive type of therapy involving a parent. Young children with depression may benefit from an interactive kind of parent-child therapy where parents are given tips on how to assist their kids understand and control their feelings. This treatment examined how a novel approach to parent-child therapy affected depressed children between the ages of 3 and 7 years old. The new therapeutic strategy was based on a commonly used parent-child therapy programme, in which a therapist advises a parent while they engage with their child, but with an additional focus on emotional growth.

(3) Psychotherapy (F_3)**:** The process of treating psychological illnesses with psychological and linguistic methods is known as psychotherapy. To support people in identifying and defeating negative thinking or behavioral patterns, the majority of psychotherapy methods encourage a therapeutic friendship between the therapist and the client. Psychotherapy is sometimes referred to as "talk therapy" since it includes a patient and a psychotherapist conversing while they are both in the same room. However, it goes far deeper than that. Psychotherapists have received professional training in a number of strategies that they use to help patients in recovering mental disorders, resolving interpersonal tensions, and developing good life changes. Different forms of psychotherapy are offered for depression. Cognitive behavioral therapy (CBT) and interpersonal therapy (IPT) are two psychotherapies that are useful in treating depression. Some patients with depression may benefit from using more traditional psychotherapies, such dynamic therapy, for a short period of time.

(4) Brain stimulation therapy (BST) (F_4)**:** When usual depressive disorders therapies have failed, brain stimulation therapy (BST) may be an alternative for certain patients. BST involves directly stimulating or inhibiting the brain using electrical or magnetic waves. Repetitive transcranial magnetic stimulation and electroconvulsive treatment are the two most popular types of BST. Other brain stimulation therapies are more recent and, in certain situations, are still in the testing phase.

(5) Electroconvulsive therapy (ECT) (F_5)**:** ECT is the most often used treatment for resistant depression. The most efficient and fastest treatment for resistant depression is ECT, which has been adjusted to eliminate all that pain previously caused by it. The negative aspect is that it could damage memory and causes brain seizures when taken. The therapeutic benefits may also diminish with time. There may be some relief from new techniques for stimulating the brain. These technologies take use of the concept that the brain is an electrical organ, reaction to electrical and magnetic stimulation to modify brain circuits and change brain activity. The graphically framework of a real-life problem dealing with the diagnosis and treatment of depression is shown in Fig. 2.

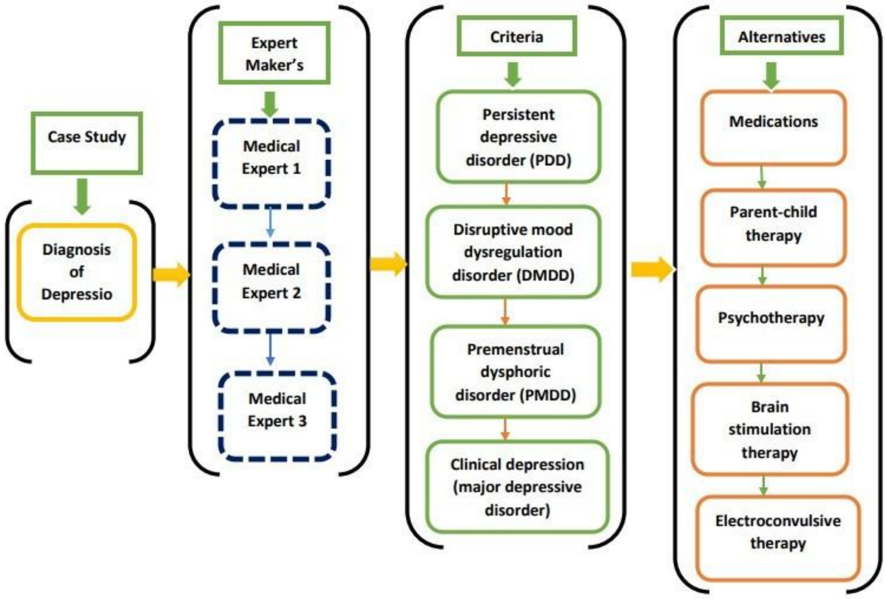

Figure 2 The schematic representation of clinical diagnosis process.

7.2 Algorithm-1

Step-1: Assuming that the five alternatives $Ғ_i (i = 1,2,..,5)$, four criteria $H_j (j = 1,2,3,4)$ with 3 decision matrices DM_k given in **Tables 1-3**, will be evaluated by three experts. All criteria are benefits types.

Table 1. Decision Matrix 1

$q = 7$	H_1	H_2
₣$_1$	$\begin{pmatrix} \langle [.8,.9] \times e^{i2\pi[0.7,0.9]} \rangle, \\ \langle [.9,.87] \times e^{i2\pi[0.8,0.9]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.78,.87] \times e^{i2\pi[.85,.85]} \rangle, \\ \langle [.9,.9] \times e^{i2\pi[.9,.85]} \rangle \end{pmatrix}$
₣$_2$	$\begin{pmatrix} \langle [.9,.8] \times e^{i2\pi[0.9,0.88]} \rangle, \\ \langle [.98,.8] \times e^{i2\pi[0.8,0.9]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.89,.8] \times e^{i2\pi[.95,.8]} \rangle, \\ \langle [.9,.9] \times e^{i2\pi[.88,.8]} \rangle \end{pmatrix}$
₣$_3$	$\begin{pmatrix} \langle [.98,.87] \times e^{i2\pi[0.9,0.87]} \rangle, \\ \langle [.95,.78] \times e^{i2\pi[0.78,0.95]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9,.9] \times e^{i2\pi[.88,.8]} \rangle, \\ \langle [.88,.8] \times e^{i2\pi[.9,.8]} \rangle \end{pmatrix}$
₣$_4$	$\begin{pmatrix} \langle [.88,.8] \times e^{i2\pi[.85,.8]} \rangle, \\ \langle [.78,.98] \times e^{i2\pi[.78,.87]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.87,.9] \times e^{i2\pi[.8,.95]} \rangle, \\ \langle [.9,.8] \times e^{i2\pi[.88,.85]} \rangle \end{pmatrix}$
₣$_5$	$\begin{pmatrix} \langle [.89,.9] \times e^{i2\pi[.9,.88]} \rangle, \\ \langle [.95,.8] \times e^{i2\pi[.85,.85]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.98,.8] \times e^{i2\pi[.9,.9]} \rangle, \\ \langle [.85,.8] \times e^{i2\pi[.88,.8]} \rangle \end{pmatrix}$

$q = 7$	H_3	H_4
₣$_1$	$\begin{pmatrix} \langle [.9,.78] \times e^{i2\pi[.9,.8]} \rangle, \\ \langle [.89,.89] \times e^{i2\pi[.9,.9]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9,.9] \times e^{i2\pi[.78,.87]} \rangle, \\ \langle [.9,.8] \times e^{i2\pi[.85,.8]} \rangle \end{pmatrix}$
₣$_2$	$\begin{pmatrix} \langle [.8,.9] \times e^{i2\pi[.8,.95]} \rangle, \\ \langle [.9,.88] \times e^{i2\pi[.88,.85]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9,.8] \times e^{i2\pi[.77,.95]} \rangle, \\ \langle [.87,.78] \times e^{i2\pi[.87,.9]} \rangle \end{pmatrix}$
₣$_3$	$\begin{pmatrix} \langle [.8,.9] \times e^{i2\pi[.88,.88]} \rangle, \\ \langle [.95,.8] \times e^{i2\pi[.98,.6]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9,.9] \times e^{i2\pi[.9,.7]} \rangle, \\ \langle [.8,.87] \times e^{i2\pi[.9,.8]} \rangle \end{pmatrix}$
₣$_4$	$\begin{pmatrix} \langle [.99,.89] \times e^{i2\pi[.7,.9]} \rangle, \\ \langle [.8,.78] \times e^{i2\pi[.8,.9]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9,.98] \times e^{i2\pi[.78,.87]} \rangle, \\ \langle [.98,.8] \times e^{i2\pi[.85,.8]} \rangle \end{pmatrix}$
₣$_5$	$\begin{pmatrix} \langle [.9,.9] \times e^{i2\pi[.9,.8]} \rangle, \\ \langle [.7,.9] \times e^{i2\pi[.77,.95]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.88,.9] \times e^{i2\pi[.89,.9]} \rangle, \\ \langle [.7,.9] \times e^{i2\pi[.95,.8]} \rangle \end{pmatrix}$

Table 2 Decision Matrix 2

$q=7$	H_1	H_2
Ꞙ$_1$	$\begin{pmatrix}\langle[.88,.9]\times e^{i2\pi[.85,.85]}\rangle,\\ \langle[.9,.8]\times e^{i2\pi[.9,.85]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.8,0.9]\times e^{i2\pi[0.7,0.9]}\rangle,\\ \langle[0.9,0.9]\times e^{i2\pi[0.8,0.9]}\rangle\end{pmatrix}$
Ꞙ$_2$	$\begin{pmatrix}\langle[.99,.89]\times e^{i2\pi[.7,.9]}\rangle,\\ \langle[.8,.85]\times e^{i2\pi[.8,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.8,.78]\times e^{i2\pi[.85,.8]}\rangle,\\ \langle[.8,.98]\times e^{i2\pi[.78,.87]}\rangle\end{pmatrix}$
Ꞙ$_3$	$\begin{pmatrix}\langle[.85,.9]\times e^{i2\pi[.77,.95]}\rangle,\\ \langle[.9,.85]\times e^{i2\pi[.87,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.98,.87]\times e^{i2\pi[.9,.87]}\rangle,\\ \langle[.95,.78]\times e^{i2\pi[.78,.95]}\rangle\end{pmatrix}$
Ꞙ$_4$	$\begin{pmatrix}\langle[.9,.89]\times e^{i2\pi[.78,.87]}\rangle,\\ \langle[.8,.88]\times e^{i2\pi[.85,.8]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.88,.98]\times e^{i2\pi[.77,.95]}\rangle,\\ \langle[.9,.85]\times e^{i2\pi[.87,.9]}\rangle\end{pmatrix}$
Ꞙ$_5$	$\begin{pmatrix}\langle[.98,.8]\times e^{i2\pi[.9,.9]}\rangle,\\ \langle[.85,.8]\times e^{i2\pi[.7,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.88]\times e^{i2\pi[.9,.9]}\rangle,\\ \langle[.77,.95]\times e^{i2\pi[.9,.7]}\rangle\end{pmatrix}$
$q=7$	H_3	H_4
Ꞙ$_1$	$\begin{pmatrix}\langle[.9,.98]\times e^{i2\pi[.9,.88]}\rangle,\\ \langle[.98,.8]\times e^{i2\pi[.8,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.98,.87]\times e^{i2\pi[.9,.87]}\rangle,\\ \langle[.95,.78]\times e^{i2\pi[.78,.95]}\rangle\end{pmatrix}$
Ꞙ$_2$	$\begin{pmatrix}\langle[.8,.9]\times e^{i2\pi[.8,.95]}\rangle,\\ \langle[.9,.8]\times e^{i2\pi[.88,.85]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.98,.87]\times e^{i2\pi[.9,.87]}\rangle,\\ \langle[.95,.78]\times e^{i2\pi[.78,.95]}\rangle\end{pmatrix}$
Ꞙ$_3$	$\begin{pmatrix}\langle[.9,.9]\times e^{i2\pi[.9,.7]}\rangle,\\ \langle[.88,.8]\times e^{i2\pi[.9,.8]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.89]\times e^{i2\pi[.7,.9]}\rangle,\\ \langle[.87,.9]\times e^{i2\pi[.8,.9]}\rangle\end{pmatrix}$
Ꞙ$_4$	$\begin{pmatrix}\langle[.89,.8]\times e^{i2\pi[.95,.8]}\rangle,\\ \langle[.9,.9]\times e^{i2\pi[.88,.8]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.87]\times e^{i2\pi[.85,.85]}\rangle,\\ \langle[.85,.8]\times e^{i2\pi[.9,.85]}\rangle\end{pmatrix}$
Ꞙ$_5$	$\begin{pmatrix}\langle[.9,.88]\times e^{i2\pi[.9,.88]}\rangle,\\ \langle[.9,.8]\times e^{i2\pi[.82,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.9]\times e^{i2\pi[.99,.89]}\rangle,\\ \langle[.9,.7]\times e^{i2\pi[.7,.9]}\rangle\end{pmatrix}$

Table 3 Decision Matrix 3

$q=7$	H_1	H_2
Ꞙ$_1$	$\begin{pmatrix}\langle[.9,.89]\times e^{i2\pi[.78,.87]}\rangle,\\ \langle[.88,.85]\times e^{i2\pi[.85,.8]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.79,.88]\times e^{i2\pi[.85,.85]}\rangle,\\ \langle[.9,.9]\times e^{i2\pi[.9,.85]}\rangle\end{pmatrix}$
Ꞙ$_2$	$\begin{pmatrix}\langle[.9,.87]\times e^{i2\pi[.85,.85]}\rangle,\\ \langle[.8,.85]\times e^{i2\pi[.9,.85]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.78,.9]\times e^{i2\pi[.7,.9]}\rangle,\\ \langle[.9,.9]\times e^{i2\pi[.8,.9]}\rangle\end{pmatrix}$
Ꞙ$_3$	$\begin{pmatrix}\langle[.9,.86]\times e^{i2\pi[.77,.95]}\rangle,\\ \langle[.85,.88]\times e^{i2\pi[.87,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.98]\times e^{i2\pi[.78,.87]}\rangle,\\ \langle[.98,.8]\times e^{i2\pi[.85,.8]}\rangle\end{pmatrix}$
Ꞙ$_4$	$\begin{pmatrix}\langle[.89,.9]\times e^{i2\pi[.7,.9]}\rangle,\\ \langle[.88,.9]\times e^{i2\pi[.8,.9]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.88]\times e^{i2\pi[.77,.95]}\rangle,\\ \langle[.78,.85]\times e^{i2\pi[.87,.9]}\rangle\end{pmatrix}$
Ꞙ$_5$	$\begin{pmatrix}\langle[.9,.9]\times e^{i2\pi[.98,.98]}\rangle,\\ \langle[.7,.9]\times e^{i2\pi[.78,.87]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.89]\times e^{i2\pi[.99,.89]}\rangle,\\ \langle[.78,.87]\times e^{i2\pi[.7,.9]}\rangle\end{pmatrix}$
$q=7$	H_3	H_4
Ꞙ$_1$	$\begin{pmatrix}\langle[.9,.88]\times e^{i2\pi[.9,.7]}\rangle,\\ \langle[.8,.89]\times e^{i2\pi[.9,.8]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.99,.89]\times e^{i2\pi[.7,.9]}\rangle,\\ \langle[.8,.8]e^{i2\pi[.8,.9]}\rangle\end{pmatrix}$
Ꞙ$_2$	$\begin{pmatrix}\langle[.8,.9]\times e^{i2\pi[.8,.95]}\rangle,\\ \langle[.9,.8]\times e^{i2\pi[.88,.85]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.96]\times e^{i2\pi[.9,.8]}\rangle,\\ \langle[.89,.89]\times e^{i2\pi[.9,.7]}\rangle\end{pmatrix}$
Ꞙ$_3$	$\begin{pmatrix}\langle[.8,.96]\times e^{i2\pi[.85,.85]}\rangle,\\ \langle[.9,.87]\times e^{i2\pi[.9,.85]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.9]\times e^{i2\pi[.9,.7]}\rangle,\\ \langle[.85,.87]\times e^{i2\pi[.9,.8]}\rangle\end{pmatrix}$
Ꞙ$_4$	$\begin{pmatrix}\langle[.79,.9]\times e^{i2\pi[.88,.88]}\rangle,\\ \langle[.95,.8]\times e^{i2\pi[.98,.6]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.98]\times e^{i2\pi[.78,.87]}\rangle,\\ \langle[.98,.8]\times e^{i2\pi[.85,.8]}\rangle\end{pmatrix}$
Ꞙ$_5$	$\begin{pmatrix}\langle[.9,.77]\times e^{i2\pi[.95,.9]}\rangle,\\ \langle[.85,.85]\times e^{i2\pi[.85,.85]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.9]\times e^{i2\pi[.9,.9]}\rangle,\\ \langle[.7,.85]\times e^{i2\pi[.7,.9]}\rangle\end{pmatrix}$

Step-2: Now we will apply the CN-LDFDWG AOs with weights $£=(0.4,0.3,0.3)^T$, to find the decision matrix (DM) **Tables 1-3** into combined aggregated matrix by applying the proposed operators, following **Table 4 and Table 5** showed the combined DEM for CN-LDFDWG and CN-LDFDOWG, similarly calculated for CN-LDFDHWG aggregation operators by means of equations 9, 14 and 16, also for different operational parameters value of $\Psi=2,3,5$ and $\Psi=10$.

Table 4 Combined decision matrix of CN-LDFDWG for $\Psi = 2$

$q = 7$	H_1	H_2
$Ꞙ_1$	$\begin{pmatrix}\langle[.844,.086]e^{i2\pi[.754,.074]}\rangle,\\ \langle[.894,.055]e^{i2\pi[.837,.069]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.789,.076]e^{i2\pi[.783,.068]}\rangle,\\ \langle[.9,.089]e^{i2\pi[.858,.068]}\rangle\end{pmatrix}$
$Ꞙ_2$	$\begin{pmatrix}\langle[.915,.063]e^{i2\pi[.792,.074]}\rangle,\\ \langle[.838,.047]e^{i2\pi[.821,.081]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.819,.060]e^{i2\pi[.796,.061]}\rangle,\\ \langle[.858,.198]e^{i2\pi[.817,.067]}\rangle\end{pmatrix}$
$Ꞙ_3$	$\begin{pmatrix}\langle[.897,.073]e^{i2\pi[.805,.138]}\rangle,\\ \langle[.894,.054]e^{i2\pi[.825,.129]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.914,.198]e^{i2\pi[.842,.056]}\rangle,\\ \langle[.916,.032]e^{i2\pi[.837,.112]}\rangle\end{pmatrix}$
$Ꞙ_4$	$\begin{pmatrix}\langle[.889,.072]e^{i2\pi[.768,.067]}\rangle,\\ \langle[.808,.210]e^{i2\pi[.803,.069]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.881,.198]e^{i2\pi[.781,.158]}\rangle,\\ \langle[.847,.047]e^{i2\pi[.874,.078]}\rangle\end{pmatrix}$
$Ꞙ_5$	$\begin{pmatrix}\langle[.909,.079]e^{i2\pi[.914,.198]}\rangle,\\ \langle[.796,.061]e^{i2\pi[.768,.071]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.919,.065]e^{i2\pi[.915,.086]}\rangle,\\ \langle[.799,.114]e^{i2\pi[.795,.059]}\rangle\end{pmatrix}$
$q = 7$	H_3	H_4
$Ꞙ_1$	$\begin{pmatrix}\langle[.9,.197]e^{i2\pi[.9,.049]}\rangle,\\ \langle[.864,.072]e^{i2\pi[.858,.079]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.933,.080]e^{i2\pi[.769,.074]}\rangle,\\ \langle[.865,.032]e^{i2\pi[.809,.116]}\rangle\end{pmatrix}$
$Ꞙ_2$	$\begin{pmatrix}\langle[.8,.089]e^{i2\pi[.8,.158]}\rangle,\\ \langle[.9,.055]e^{i2\pi[.88,.054]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.914,.132]e^{i2\pi[.828,.123]}\rangle,\\ \langle[.893,.054]e^{i2\pi[.839,.117]}\rangle\end{pmatrix}$
$Ꞙ_3$	$\begin{pmatrix}\langle[.821,.138]e^{i2\pi[.875,.057]}\rangle,\\ \langle[.908,.047]e^{i2\pi[.919,.037]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.086]e^{i2\pi[.798,.057]}\rangle,\\ \langle[.832,.074]e^{i2\pi[.858,.061]}\rangle\end{pmatrix}$
$Ꞙ_4$	$\begin{pmatrix}\langle[.862,.075]e^{i2\pi[.779,.074]}\rangle,\\ \langle[.853,.059]e^{i2\pi[.851,.065]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.9,.235]e^{i2\pi[.797,.062]}\rangle,\\ \langle[.911,.033]e^{i2\pi[.862,.041]}\rangle\end{pmatrix}$
$Ꞙ_5$	$\begin{pmatrix}\langle[.9,0.073]e^{i2\pi[.911,.069]}\rangle,\\ \langle[.773,0.069]e^{i2\pi[.804,.126]}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.891,.089]e^{i2\pi[.909,.086]}\rangle,\\ \langle[.733,.068]e^{i2\pi[.749,.075]}\rangle\end{pmatrix}$

Similarly, **Table 5** is for CN-LDFDOWG aggregation operator**.**

Table. 5 Combined decision matrix of CN-LDFDOWG for $\Psi = 2$

$q = 7$	H_1	H_2
$₣_1$	$\begin{pmatrix} \langle[.9,.197]e^{i2\pi[.9,.049]}\rangle, \\ \langle[.864,.072]e^{i2\pi[.858,.079]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.933,.0804]e^{i2\pi[.769,.074]}\rangle, \\ \langle[.865,.032]e^{i2\pi[.809,.116]}\rangle \end{pmatrix}$
$₣_2$	$\begin{pmatrix} \langle[.914,.132]e^{i2\pi[0.828,0.123]}\rangle, \\ \langle[.893,.054]e^{i2\pi[0.839,0.117]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.8,.089]e^{i2\pi[.8,.158]}\rangle, \\ \langle[.9,.055]e^{i2\pi[.88,.054]}\rangle \end{pmatrix}$
$₣_3$	$\begin{pmatrix} \langle[.821,.138]e^{i2\pi[0.875,0.057]}\rangle, \\ \langle[.908,.047]e^{i2\pi[0.919,0.037]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.914,.198]e^{i2\pi[.842,.056]}\rangle, \\ \langle[.916,.032]e^{i2\pi[.837,.112]}\rangle \end{pmatrix}$
$₣_4$	$\begin{pmatrix} \langle[.9,.235]e^{i2\pi[0.797,0.062]}\rangle, \\ \langle[.911,.033]e^{i2\pi[0.862,0.041]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.881,.198]e^{i2\pi[.781,.158]}\rangle, \\ \langle[.847,.047]e^{i2\pi[.874,.078]}\rangle \end{pmatrix}$
$₣_5$	$\begin{pmatrix} \langle[.919,.065]e^{i2\pi[.915,.086]}\rangle, \\ \langle[.799,.114]e^{i2\pi[.795,.059]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.9,.073]e^{i2\pi[.911,.069]}\rangle, \\ \langle[.772,.069]e^{i2\pi[.804,.126]}\rangle \end{pmatrix}$
$q = 7$	H_3	H_4
$₣_1$	$\begin{pmatrix} \langle[.789,.076]e^{i2\pi[.783,.068]}\rangle, \\ \langle[.9,.089]e^{i2\pi[.858,.068]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.844,.086]e^{i2\pi[.754,.074]}\rangle, \\ \langle[.894,.055]e^{i2\pi[.837,.069]}\rangle \end{pmatrix}$
$₣_2$	$\begin{pmatrix} \langle[.915,.063]e^{i2\pi[.792,.074]}\rangle, \\ \langle[.838,.047]e^{i2\pi[.821,.081]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.819,.06]e^{i2\pi[.796,.061]}\rangle, \\ \langle[.8578,.198]e^{i2\pi[.817,.067]}\rangle \end{pmatrix}$
$₣_3$	$\begin{pmatrix} \langle[.897,.073]e^{i2\pi[.805,.138]}\rangle, \\ \langle[.894,.054]e^{i2\pi[.825,.129]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.9,.086]e^{i2\pi[.798,.057]}\rangle, \\ \langle[.832,.074]e^{i2\pi[.858,.061]}\rangle \end{pmatrix}$
$₣_4$	$\begin{pmatrix} \langle[.862,.075]e^{i2\pi[.779,.074]}\rangle, \\ \langle[.853,.059]e^{i2\pi[.851,.065]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.889,.072]e^{i2\pi[.768,.067]}\rangle, \\ \langle[.808,.210]e^{i2\pi[.803,.069]}\rangle \end{pmatrix}$
$₣_5$	$\begin{pmatrix} \langle[.909,.079]e^{i2\pi[.914,.198]}\rangle, \\ \langle[.796,.061]e^{i2\pi[.768,.0709]}\rangle \end{pmatrix}$	$\begin{pmatrix} \langle[.891,.089]e^{i2\pi[.909,.086]}\rangle, \\ \langle[.733,.068]e^{i2\pi[.749,.075]}\rangle \end{pmatrix}$

Step-3: Again, calculated the combined decision matrix (**Table 4 and 5)** with weights $£ = \{.35,.3,.25,.1\}$ by means of applying equation 9, 14 and 16 of CN-LDFDWG AOs.

Step-4: Then we find the expectation score function (ESf) of combined matrix presented in step-3. **Table 6** represented the ESf values by means of CN-LDFDWG, CN-LDFDOWG and CN-LDFDHWG AOs for $\Psi = 2,3,5$ and 10.

Table 6. ES_f values of Algorithm 1

$q = 7, \Psi = 2$	$ES_f(₣_1)$	$ES_f(₣_2)$	$ES_f(₣_3)$	$ES_f(₣_4)$	$ES_f(₣_5)$
CN-LDFDWG	.69733	.69299	.71741	.69189	.71532
CN-LDFDOWG	.761	.7145	.71981	.7225	.71544
CN-LDFDHWG	.7957	.79385	.81271	.79112	.7717
$\Psi = 3$	$ES_f(₣_1)$	$ES_f(₣_2)$	$ES_f(₣_3)$	$ES_f(₣_4)$	$ES_f(₣_5)$
CN-LDFDWG	.68462	.6882	.7975	.68234	.7253
CN-LDFDOWG	.68647	.68876	.797	.6971	.7432
CN-LDFDHWG	.74546	.74468	.76134	.7438	.747
$\Psi = 5$	$ES_f(₣_1)$	$ES_f(₣_2)$	$ES_f(₣_3)$	$ES_f(₣_4)$	$ES_f(₣_5)$
CN-LDFDWG	.6758	.67291	.7829	.67599	.6929
CN-LDFDOWG	.6762	.67957	.7848	.68357	.69274
CN-LDFDHWG	.71241	.7136	.72611	.7136	.72525
$q = 7, \Psi = 1$	$ES_f(₣_1)$	$ES_f(₣_2)$	$ES_f(₣_3)$	$ES_f(₣_4)$	$ES_f(₣_5)$
CN-LDFDWG	.66251	.66134	.7532	.66334	.67452
CN-LDFDOWG	.66384	.66571	.7548	.66884	.67423
CN-LDFDHWG	.67964	.6793	.71285	.6848	.68827

S**tep-5: Table 7** identified the proposed model's ranking.

Table 7. Proposed Algorithm-1 ranking

Proposed $\Psi = 2$	Alternatives ranking
CN-LDFDWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_1 > Ꞙ_2 > Ꞙ_4$
CN-LDFDOWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_2 > Ꞙ_4 > Ꞙ_1$
CN-LDFDHWG	$Ꞙ_3 > Ꞙ_1 > Ꞙ_2 > Ꞙ_4 > Ꞙ_5$
	$\Psi = 3$
CN-LDFDWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_1 > Ꞙ_4 > Ꞙ_2$
CN-LDFDOWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_2 > Ꞙ_1$
CN-LDFDHWG	$Ꞙ_3 > Ꞙ_1 > Ꞙ_2 > Ꞙ_4 > Ꞙ_5$
	$\Psi = 5$
CN-LDFDWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_1 > Ꞙ_2$
CN-LDFDOWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_2 > Ꞙ_1$
CN-LDFDHWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_2 > Ꞙ_1$
	$\Psi = 1$
CN-LDFDWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_1 > Ꞙ_2$
CN-LDFDOWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_2 > Ꞙ_1$
CN-LDFDHWG	$Ꞙ_3 > Ꞙ_5 > Ꞙ_4 > Ꞙ_1 > Ꞙ_2$

Step-6: Our conclusion from **Table 7** is $Ꞙ_3$ i.e. **Psychotherapy**, is the optimal (alternative) diagnosis treatment for depression. Because they have received professional training techniques, psychotherapists can support patients in recovering from mental health conditions, handling personal problems, and implementing good changes in their lives. No matter what kind of therapy you select, psychotherapy should be a pleasant and supportive experience. You should always feel comfortable discussing your feelings and problems with depression when dealing with a psychotherapist. The suggested algorithm-1's graphic rating is displayed in below Fig. 3.

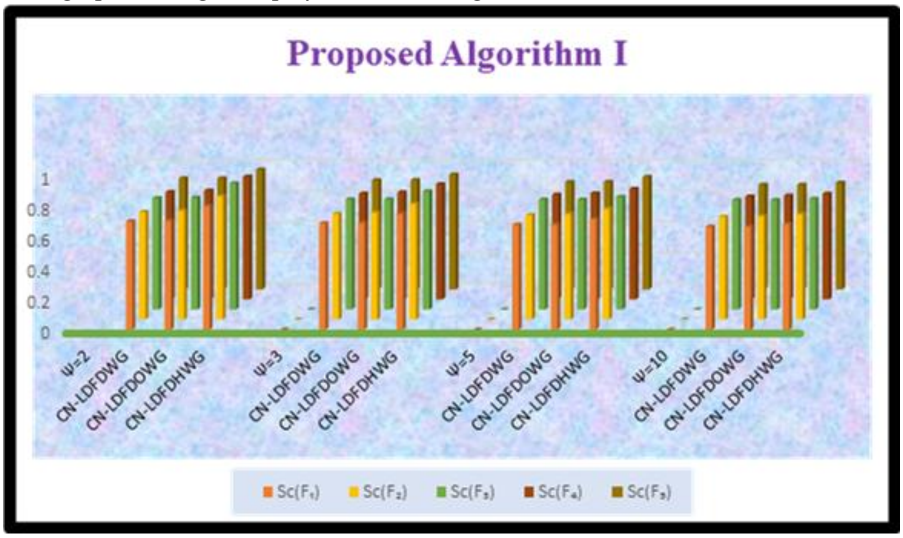

Figure 3 Graphically ranking of proposed algorithm-I

7.3 Algorithm-2: CODAS model

Step-1: Decision matrices of CN-LDFNs are presented in **Table 1-3**, with 05 $(Ꞙ_i = 1-5)$ alternatives according to 04 $(H_j = 1-4)$ criteria with three expert matrices.

Step-2: We evaluated the weighted normalized decision matrix (WNDM) by using equation (21), following **Table 8** listed the result.

Table 8 Weighted normalized (WNDM) of CN-LDFDWG for $\Psi = 2$

$q = 7$	H_1	H_2
$Ғ_1$	$\begin{pmatrix} \langle [.844, .086] e^{i2\pi[.754,.074]} \rangle, \\ \langle [.894, .055] e^{i2\pi[.837,.069]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.789, .076] e^{i2\pi[.783,.068]} \rangle, \\ \langle [.9, .089] e^{i2\pi[.858,.068]} \rangle \end{pmatrix}$
$Ғ_2$	$\begin{pmatrix} \langle [.915, .063] e^{i2\pi[.792,.074]} \rangle, \\ \langle [.838, .047] e^{i2\pi[.821,.081]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.819, .060] e^{i2\pi[.796,.061]} \rangle, \\ \langle [.858, .198] e^{i2\pi[.817,.067]} \rangle \end{pmatrix}$
$Ғ_3$	$\begin{pmatrix} \langle [.897, .073] e^{i2\pi[.805,.138]} \rangle, \\ \langle [.894, .054] e^{i2\pi[.825,.129]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.914, .198] e^{i2\pi[.842,.056]} \rangle, \\ \langle [.916, .032] e^{i2\pi[.837,.112]} \rangle \end{pmatrix}$
$Ғ_4$	$\begin{pmatrix} \langle [.889, .072] e^{i2\pi[.768,.067]} \rangle, \\ \langle [.808, .210] e^{i2\pi[.803,.069]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.881, .198] e^{i2\pi[.781,.158]} \rangle, \\ \langle [.847, .047] e^{i2\pi[.874,.078]} \rangle \end{pmatrix}$
$Ғ_5$	$\begin{pmatrix} \langle [.909, .079] e^{i2\pi[.914,.198]} \rangle, \\ \langle [.796, .061] e^{i2\pi[.768,.071]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.919, .065] e^{i2\pi[.915,.086]} \rangle, \\ \langle [.799, .114] e^{i2\pi[.795,.059]} \rangle \end{pmatrix}$
$q = 7$	H_3	H_4
$Ғ_1$	$\begin{pmatrix} \langle [.9, .197] e^{i2\pi[.9,.049]} \rangle, \\ \langle [.864, .072] e^{i2\pi[.858,.079]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.933, .080] e^{i2\pi[.769,.074]} \rangle, \\ \langle [.865, .032] e^{i2\pi[.809,.116]} \rangle \end{pmatrix}$
$Ғ_2$	$\begin{pmatrix} \langle [.8, .089] e^{i2\pi[.8,.158]} \rangle, \\ \langle [.9, .055] e^{i2\pi[.88,.054]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.914, .132] e^{i2\pi[.828,.123]} \rangle, \\ \langle [.893, .054] e^{i2\pi[.839,.117]} \rangle \end{pmatrix}$
$Ғ_3$	$\begin{pmatrix} \langle [.821, .138] e^{i2\pi[.875,.057]} \rangle, \\ \langle [.908, .047] e^{i2\pi[.919,.037]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9, .086] e^{i2\pi[.798,.057]} \rangle, \\ \langle [.832, .074] e^{i2\pi[.858,.061]} \rangle \end{pmatrix}$
$Ғ_4$	$\begin{pmatrix} \langle [.862, .075] e^{i2\pi[.779,.074]} \rangle, \\ \langle [.853, .059] e^{i2\pi[.851,.065]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.9, .235] e^{i2\pi[.797,.062]} \rangle, \\ \langle [.911, .033] e^{i2\pi[.862,.041]} \rangle \end{pmatrix}$
$Ғ_5$	$\begin{pmatrix} \langle [.9, 0.073] e^{i2\pi[.911,.069]} \rangle, \\ \langle [.773, 0.069] e^{i2\pi[.804,.126]} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [.891, .089] e^{i2\pi[.909,.086]} \rangle, \\ \langle [.733, .068] e^{i2\pi[.749,.075]} \rangle \end{pmatrix}$

Step-4: By applying equation 23, **Table 9** showed the negative ideal solution (NIS) for $\Psi = 2$ in the form of CN-LDFNs based on Dombi geometric operators.

Table 9 NIS by CN-LDFDWG for $\Psi = 2$

$q=7$	H_1	H_2
$Ғ_1$	$\left(\begin{array}{c}\langle[.8438,.0626]e^{i2\pi[.7538,.0671]}\rangle, \\ \langle[.7962,.0472]e^{i2\pi[.7678,.0689]}\rangle\end{array}\right)$	$\left(\begin{array}{c}\langle[.7887,.0601]e^{i2\pi[.7811,.0562]}\rangle, \\ \langle[.7993,.0315]e^{i2\pi[.7953,.0592]}\rangle\end{array}\right)$
$q=7$	H_3	H_3
$Ғ_1$	$\left(\begin{array}{c}\langle[.8,.0731]e^{i2\pi[.7797,.0486]}\rangle, \\ \langle[.773,.0469]e^{i2\pi[.8039,.0365]}\rangle\end{array}\right)$	$\left(\begin{array}{c}\langle[.8914,.0804]e^{i2\pi[.7686,.0570]}\rangle, \\ \langle[.7333,.0315]e^{i2\pi[.7498,.0411]}\rangle\end{array}\right)$

Step-5: The E_i and T_i distances of $Ғ_i$ obtained from NIS by applying equation 24 and equation 25 are appeared in **Table 10(a)**.

Table 10(a) Distances of CN-LDFDWG

Alternatives	Euclidean (E_i)	Taxicab (T_i)
$Ғ_1$	.338654834	1.346596614
$Ғ_2$	.360650535	1.449995223
$Ғ_3$	.391944044	1.696415008
$Ғ_4$	.400249224	1.517588522
$Ғ_5$	.387345059	1.384885548

Similarly, **Table 10(b)** listed for CN-LDFDOWG.

Table 10(b) Distances of CN-LDFDOWG

Alternatives	Euclidean (E_i)	Taxicab (T_i)
$Ғ_1$	.36312814	1.420271405
$Ғ_2$	.367247115	1.523671997
$Ғ_3$	.403955641	1.770093949
$Ғ_4$	.38716735	1.591267593
$Ғ_5$	.388424669	1.458561662

Table 10(c) listed for CN-LDFDHWG.

Table 10(c) Distances of CN-LDFDHWG

Alternatives	Euclidean (E_i)	Taxicab (T_i)
$Ғ_1$	.182035704	.580138584
$Ғ_2$	.162678209	.55376891
$Ғ_3$	.204706387	.75188223
$Ғ_4$	.154194972	.510605739
$Ғ_5$	.20096412	.543656257

Step-6: **Table 11** displayed the relative assessment (ReAs) matrix for CN-LDFDWG by applying equation 27. Similarly, we calculated for CN-LDFDOWG and CN-LDFDHWG operators.

Table 11 ReAs matrix of CN-LDFDWG

	$₣_1$	$₣_2$	$₣_3$	$₣_4$	$₣_5$
$₣_1$	0	−.02195	−.05292	−.06138	−.04865
$₣_2$	.02204	0	−.03114	−.03955	−.02673
$₣_3$	.05366	.03145	0	−.00834	.00463
$₣_4$	.06181	.03965	.00828	0	.01294
$₣_5$	.04873	.02666	−.00457	−.01287	0

Step-7: The assessment score (ASs) values of alternatives is listed in the following **Table 12** by using CN-LDFDWG operators by choosing different parameters $\Psi = 2, 3, 5$ and 10 by applying equation 29.

Table 12 Assessment score values for different parameters $\Psi's$

$q = 7, \Psi = 2$	$AS(₣_1)$	$AS(₣_2)$	$AS(₣_3)$	$AS(₣_4)$	$AS(₣_5)$
$CN-LDFDWG$	−0.18490	−0.07537	0.12267	0.08140	0.05795
$CN-LDFDOWG$	−0.09389	−0.07349	0.11048	0.02608	0.03229
$CN-LDFDHWG$	0.00571	−0.09101	0.11946	−0.13328	0.10027
$q = 7, \Psi = 3$	$AS(₣_1)$	$AS(₣_2)$	$AS(₣_3)$	$AS(₣_4)$	$AS(₣_5)$
$CN-LDFDWG$	−0.16047	−0.06255	0.08729	0.08115	0.05615
$CN-LDFDOWG$	−0.08419	−0.06824	0.12006	0.00311	0.03075
$CN-LDFDHWG$	0.01017	−0.09531	0.15906	−0.17163	0.09943
$q = 7, \Psi = 5$	$AS(₣_1)$	$AS(₣_2)$	$AS(₣_3)$	$AS(₣_4)$	$Ac(₣_5)$
$CN-LDFDWG$	−0.15469	−0.04264	0.06979	0.06654	0.06247
$CN-LDFDOWG$	−0.11544	−0.06533	0.09867	0.07154	0.01227
$CN-LDFDHWG$	0.02613	−0.10464	0.18613	−0.20921	0.10369
$q = 7, \Psi = 10$	$AS(₣_1)$	$AS(₣_2)$	$AS(₣_3)$	$AS(₣_4)$	$AS(₣_5)$
$CN-LDFDWG$	−0.16128	−0.02148	0.08708	0.05157	0.04542
$CN-LDFDOWG$	−0.10388	−0.09229	0.11357	0.04369	0.04057
$CN-LDFDHWG$	0.01502	−0.07284	0.17505	−0.23200	0.11681

Step-8: We ranked the alternatives on the base of highest values of ASs_i, so $₣_3$ is the optimal alternative.

Table 13 Ranking of CODAS method for CN-LDFDWG

$\Psi = 2$	Ranking for different $\Psi's$
CN-LDFDWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDOWG	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$
$\Psi = 3$	
CN-LDFDWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDOWG	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$
$\Psi = 5$	
CN-LDFDWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDOWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$
$\Psi = 10$	
CN-LDFDWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDOWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$

We obtained the conclusion from the ranking of above **Table 13** is $Ғ_3$ i.e. **Psychotherapy**, is the optimal diagnosis treatment for depression. Psychotherapists can support patients in recovering from mental health conditions, handling personal problems, and implementing good changes in their lives. No matter what kind of therapy you select, psychotherapy should be a pleasant and supportive experience. You should always feel comfortable discussing your feelings and problems with depression when dealing with a psychotherapist. Following Fig.4 showed the graphic rating of proposed CODAS algorithm II.

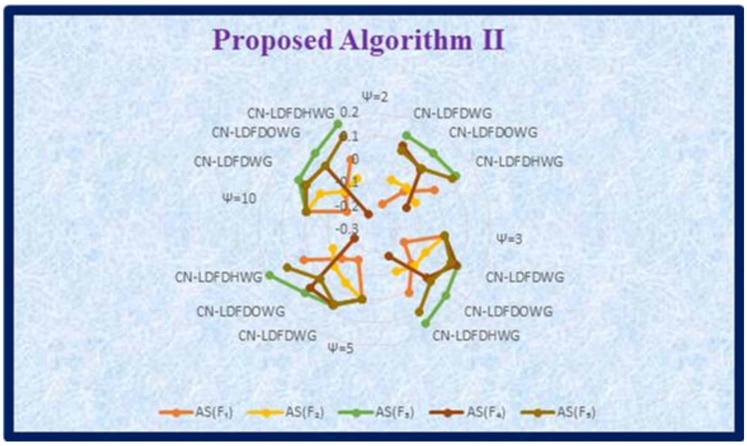

Figure 4 Graphically ranking of proposed algorithm-II

8. Comparison Analysis

For the accuracy of the suggested technique, four (04) existing operators LDFS, q-RLDFS, CLDFS and CN-LDFS are taken into consideration for comparison. We decided to use algorithm-1 for the comparative section. As can be shown, the suggested work is stable and feasible because the optimal choice are the same by utilizing the Algorithm-1 for existing approach. The suggested approach is compared with LDFS in Section 8.1. Section 8.2 then presented a comparison using q-RLDFS. Section 8.3 went on to compare it with CLDFS. Section 8.4 included a comparison with CN-LDFS.

8.1 Comparison with LDF operators [40]

Here, we compared the planned work with LDFS [40]. We used LDFOWG, LDFHWG, and LDFWG aggregation operations to the input data from **Tables 1-3**. The ES_f value of LDF operators is listed in **Table 14**.

Table 14 ES_f values of LDF operators

	ESF value of alternatives				
Existing	$ES_f(₣_1)$	$ES_f(₣_2)$	$ES_f(₣_3)$	$ES_f(₣_4)$	$ES_f(₣_5)$
LDFWG	.49618	.50031	.51569	.48870	.49875
LDFOWG	.46286	.49457	.50736	.45463	.49056
LDFHWG	.51256	.47632	.52065	.46375	.50062

The following **Table 15** identifies the ranking of LDF technique.

Table 15 Ranking of LDF Operator

Existing Operators	Ranking
LDFWG [40]	$₣_3>₣_2>₣_5>₣_1>₣_4$
LDFOWG [40]	$₣_3>₣_2>₣_5>₣_1>₣_4$
LDFHWG [40]	$₣_3>₣_1>₣_5>₣_2>₣_4$

According to the rating of existing LDFWG operators above, $₣_3$ is the most effective alternative; this is exactly same to the recommended approach obtained from both algorithms. From this ranking we again obtained the Psychotherapy the best diagnosed technique for depression.

8.2 Comparison with q-RLDF operators [47]

Here, we compared the planned work with q-RLDFS [47]. We used q-RLDFWG operators to the input data given in **Tables 1-3**. The ES_f value of q-RLDF operators is listed in **Table 16**.

Table 16 ES_f values of q-RLDF AOs

Existing	$ES_f(₣_1)$	$ES_f(₣_2)$	$ES_f(₣_3)$	$ES_f(₣_4)$	$ES_f(₣_5)$
q-RLDFWG	.50819	.49382	.55296	.46329	.47592
q-RLDFOWG	.51062	.48832	.54635	.44657	.48561
q-RLDFHWG	.49342	.47235	.49965	.40835	.49563

The following **Table 17** identifies the ranking of q-RLDF technique.

Table 17 Ranking of q-RLDF Operator

Existing Operators	Ranking
q-RLDFWG [47]	$₣_3>₣_1>₣_2>₣_5>₣_4$
q-RLDFOWG [47]	$₣_3>₣_1>₣_2>₣_5>₣_4$
q-RLDFHWG [47]	$₣_3>₣_5>₣_1>₣_2>₣_4$

According to the rating of existing q-RLDFWG operators above, $₣_3$ is the most effective alternative; this is exactly same to the recommended approach obtained from both algorithms. From this ranking we again obtained the **Psychotherapy** the best diagnosed technique for depression. Since the existing operators don't have complex notions, the similarity of the ranking result with the suggested technique offers us a better understanding of CV-MF, CV-NMF, and CV-RPs than q-RLDF data.

8.3 Comparison with Complex LDF operators [41]

In this paragraph, we compared the planned work with CLDFS [41]. We applied CLDFWG, CLDFOWG, and CLDFHWG aggregation operations to the input data given in **Tables 1-3**. The ESF value of existing CLDF operators is listed in **Table 18**.

Table 18 ES_f values of CLDF AOs

Existing method	$ES_f(Ғ_1)$	$ES_f(Ғ_2)$	$ES_f(Ғ_3)$	$ES_f(Ғ_4)$	$ES_f(Ғ_5)$
CLDFWG	0.48446	0.48208	0.50714	0.46363	0.50250
CLDFOWG	0.47326	0.43265	0.49978	0.46635	0.49936
CLDFHWG	0.49654	0.50013	0.51635	0.48356	0.51356

Table 19 identifies the ranking of CLDF technique.

Table 19 Ranking of CLDF Operator

Existing Operators	Ranking
CLDFWG [41]	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$
CLDFOWG [41]	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_4 > Ғ_2$
CLDFHWG [41]	$Ғ_3 > Ғ_5 > Ғ_2 > Ғ_1 > Ғ_4$

According to the rating of existing CLDFWG operators above, $Ғ_3$ is the most effective alternative; this is exactly same to the recommended approach obtained from both algorithms. From this ranking we again obtained the **Psychotherapy** the best diagnosed technique for depression. Since the existing operators don't have complex reference parameters notions, the similarity of the ranking result with the suggested technique offers us a better understanding of CV-RPs than CLDF information.

8.4 Comparison with Complex N-LDF operators [49]

In this paragraph, we compared the planned work with CN-LDFS [49]. We applied CN-LDFWG operators on **Tables 1-3**. The ES_f value of existing CN-LDF operators is listed in **Table 20**.

Table 20 ES_f of CN-LDF

Existing	$ES_f(Ғ_1)$	$ES_f(Ғ_2)$	$ES_f(Ғ_3)$	$ES_f(Ғ_4)$	$ES_f(Ғ_5)$
CN-LDFWG	.45721	.44749	.46819	.45586	.43562
CN-LDFOWG	.45986	.44845	.48484	.46561	.44324
CN-LDFHWG	.48214	.47239	.50338	.47886	.46542

The following **Table 21** identifies the ranking of CN-LDF technique.

Table 21. CN-LDF ranking

Existing method	Ranking
CN-LDFWG	$Ғ_3 > Ғ_1 > Ғ_4 > Ғ_2 > Ғ_5$
CN-LDFOWG	$Ғ_3 > Ғ_4 > Ғ_1 > Ғ_2 > Ғ_5$
CN-LDFHWG	$Ғ_3 > Ғ_1 > Ғ_4 > Ғ_2 > Ғ_5$

Since existing operators lack an idea of Dombi operators, the similarity of the ranking result with the suggested technique offers us a better understanding of CV-functions under Dombi Norms than CN-LDF information. **Table 22** presents the final and comprehensive ranking of both proposed and existing methodologies, as described below.

Table 22 Comprehensive ranking of developed and previous operators

Proposed Method		
Algorithm-I	$\Psi = 2$	$\Psi = 3$
CN-LDFDWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_4 > Ғ_2$
CN-LDFDOWG	$Ғ_3 > Ғ_5 > Ғ_2 > Ғ_4 > Ғ_1$	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_1 > Ғ_2 > Ғ_4 > Ғ_5$	$Ғ_3 > Ғ_1 > Ғ_2 > Ғ_4 > Ғ_5$
Algorithm-I	$\Psi = 5$	$\Psi = 10$
CN-LDFDWG	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_1 > Ғ_2$	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_1 > Ғ_2$
CN-LDFDOWG	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_1 > Ғ_2$
Algorithm-II (CODAS)	$\Psi = 2$	$\Psi = 3$
CN-LDFDWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDOWG	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$	$Ғ_3 > Ғ_5 > Ғ_4 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$
Algorithm-II (CODAS)	$\Psi = 5$	$\Psi = 10$
CN-LDFDWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDOWG	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$	$Ғ_3 > Ғ_4 > Ғ_5 > Ғ_2 > Ғ_1$
CN-LDFDHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$
Existing Methods		
LDFWG	$Ғ_3 > Ғ_2 > Ғ_5 > Ғ_1 > Ғ_4$	
LDFOWG	$Ғ_3 > Ғ_2 > Ғ_5 > Ғ_1 > Ғ_4$	
LDFHWG	$Ғ_3 > Ғ_1 > Ғ_5 > Ғ_2 > Ғ_4$	
q-RLDFWG	$Ғ_3 > Ғ_1 > Ғ_2 > Ғ_5 > Ғ_4$	
q-RLDFOWG	$Ғ_3 > Ғ_1 > Ғ_2 > Ғ_5 > Ғ_4$	
q-RLDFHWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$	
CLDFWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_2 > Ғ_4$	
CLDFOWG	$Ғ_3 > Ғ_5 > Ғ_1 > Ғ_4 > Ғ_2$	
CLDFHWG	$Ғ_3 > Ғ_5 > Ғ_2 > Ғ_1 > Ғ_4$	
CN-LDFWG	$Ғ_3 > Ғ_1 > Ғ_4 > Ғ_2 > Ғ_5$	
CN-LDFOWG	$Ғ_3 > Ғ_4 > Ғ_1 > Ғ_2 > Ғ_5$	
CN-LDFHWG	$Ғ_3 > Ғ_1 > Ғ_4 > Ғ_2 > Ғ_5$	

We concluded that; $Ғ_3$ (**Psychotherapy)** is chose the best treatment technique for depression. We got the identical ranking from both the developed and previous approach as listed in **Table 22,** shows the reliability of proposed method. Psychotherapists can support patients in recovering from mental health conditions, handling personal problems, and implementing good changes in their lives. No matter what kind of therapy you select, psychotherapy should be a pleasant and supportive experience. You should always feel comfortable discussing your feelings and problems with depression when dealing with a psychotherapist. Graphically ranking of existing operator is represented by Fig.5.

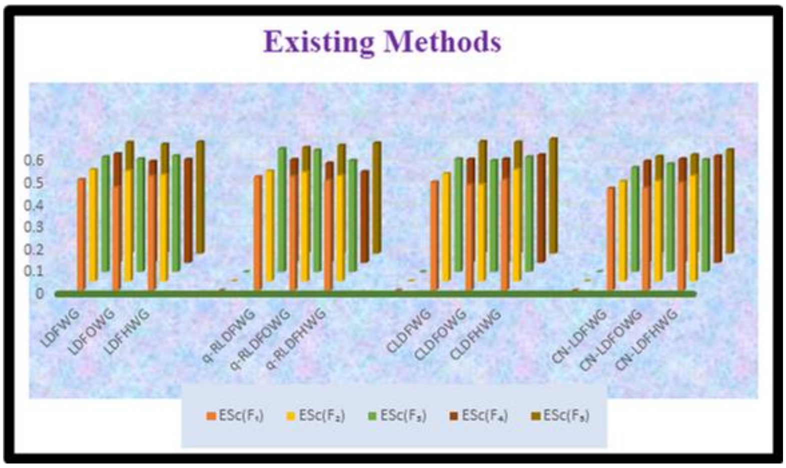

Figure 5 Graphically ranking of existing method

9. CONCLUSION

This work proposed a CN-LDF model under Dombi geometric AOs consisting of CODAS technique mapped on MCGDM problem. Using the CN-LDF Dombi rule-based operator, we have presented a model design for the diagnosis of depression illnesses. The present researched information suggests that psychotherapy is the most effective treatment for depression illness. According to several research, when taken alone, it can be more effective than medicine and have more durable good benefits. We shall study both old and modern depression treatments. There has been discussion of the five fundamental therapeutic techniques. We found out that psychotherapy treatment is successful using numerical research. The MCGDM analysis provided the proof supporting the idea of using psychotherapy to treat depression.

In practical MCGDM problems, it is important to have unrestricted flexibility in choosing the degrees of complex-valued MF (CV-MF), CV-NMF, and CV-RPs. Therefore, we adopted Dombi operators for CN-LDFNs and established the notion of CN-LDFS as an improvement of CLDFS. Compared to other approaches, the CN-LDFNs based on Dombi operators are more flexible and effective due to the inclusion of CV-RPs. Since it provides individuals a flexible and useful way to present information and make decisions, the Dombi-CN-LDFNs are being utilized successfully to solve a range of decision-making problems. Further we applied the Dombi Norms for CN-LDFNs from geometric point of view and developed an innovative systematic modification of AOs such as CN-LDFDWG, CN-LDFDOWG and CN-LDFDHWG aggregation operators. Finally, we developed two algorithms for CN-LDFS-based on Dombi geometric operators with a case study regarding to the selection of best treatment therapy for the diagnosis of depression. Finally, comparisons between proposed and existing methods for the selection of depression treatments will be reviewed showing the treatment that would be considered best practice. A number of studies and results obtained from the above Algorithms have concluded that **Psychotherapy** is effective in the treatment of depression.

In future this work can be extended to develop a power, neutral, Einstein, Dombi, Maclaurin's, and Heronian aggregation operators for CN-LDF.

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