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A Novel Decision Making Model for Selection Procedure of Artificial Intelligence Robot Through the Hyperbolic Fuzzy Soft Set

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ABSTRACT

In this study we conduct a detail of examination of hyperbolic fuzzy set. In prior research several approaches related to parabolic fuzzy set have been discussed. To define hyperbolic fuzzy set the method used for hyperbolic fuzzy soft set must be modified. Soft set theory has recently been developed as a flexible mathematical tool for handling imprecise data and uncertainty. Fuzzy soft set which combines fuzzy set theory and soft set theory further enhance this capability. This study introduces hyperbolic fuzzy soft set which use hyperbolic member ship function to improve the modeling of complex uncertainty and ambiguity. The fundamental definitions properties and operations of hyperbolic fuzzy soft set are established including union, intersection, complement and Einstein aggregation operator. Illustrative example demonstrates the effectiveness of hyperbolic fuzzy soft set and decision making and data analysis applications. The proposed framework offers an innovative strategy for dealing with multi criteria decision making problem were hyperbolic functional from better represented uncertainty.

Keywords

Hyperbolic fuzzy soft set, Decision making model, EDAS method, robot selection.

1. Introduction

The theory of soft sets, introduced by Molodtsov in 1999, provides a general mathematical framework for handling uncertainty without the restrictions inherent in classical parameterization methods. To enhance its capability, fuzzy set theory was integrated with soft sets, leading to the development of fuzzy soft sets. This hybrid structure combines the parameter-based nature of soft sets with the membership function concept of fuzzy sets, enabling a more flexible representation of vague and imprecise information. Over the years, fuzzy soft set theory has evolved significantly and gained widespread attention in uncertainty modeling and decision-making analysis. Researchers have proposed various extensions, including intuitionistic fuzzy soft sets, interval-valued fuzzy soft sets, Pythagorean fuzzy soft sets, and q-rung orthopair fuzzy soft sets. These generalizations aim to capture additional aspects of uncertainty, such as non-membership and hesitation degrees, thereby providing a more comprehensive and realistic modeling environment. In multi-criteria decision-making (MCDM), fuzzy soft sets have been effectively employed to address problems involving ambiguous and incomplete information. Different aggregation operators, similarity measures, distance functions, and ranking approaches have been developed within the fuzzy soft framework, and such methodologies have been successfully applied in diverse domains, including engineering design, medical diagnosis, supplier selection, risk evaluation, robotics assessment, and artificial intelligence systems. Recent research trends emphasize hybridization and model enhancement to improve computational efficiency and decision accuracy. The integration of fuzzy soft sets with advanced decision-making techniques such as TOPSIS, VIKOR, entropy-based weighting methods, and neural network models has strengthened their practical applicability in solving complex real-world problems. Despite these developments, there remains scope for more generalized and nonlinear structures capable of representing complex uncertainty patterns. Motivated by these research gaps, the present study introduces a novel framework based on the hyperbolic fuzzy soft set to improve the selection procedure of artificial intelligence robots, thereby contributing to both theoretical advancement and practical decision-making applications.

Decision-making based on multiple criteria (MCDM) [1-2] and decision-making based on multiple criteria MCDM. MCDM challenges vary from MCDM problems, which entail designing a "best" alternative by taking into account the trade-offs between a collection of interacting design constraints. MCDM relates to making decisions between several courses of action in the face of many, generally contradictory, qualities. In MCDM challenges, the number of choices is efficiently unlimited, and trade-off between design requirements are usually expressed as continuous functions. MCDM best known type of making decisions. It is a subset of a broader category of models of the study of operations the given address choice problems in the appearance of multiple decision criteria. The MCDM technique needs that decision alternatives be chosen (selected) based on their qualities. MCDM challenges are supposed to have a fixed, finite number of choice possibilities. MCDM techniques can be thought of as alternate strategies for integrating the data in a decision matrix for an issue with more data from the decision-maker to reach a definitive selection, screening, or ranking among the possibilities. Most MCDM approaches require extra information from the decision maker beyond the decision matrix to make choices on a final decision-making, screening, or rank. The MCDM technique, unlike the MCDM approach, does not provide decision alternatives. Instead, MCDM offers a mathematical framework for developing a collection of decision options. After identifying alternatives, they are evaluated based on how well they align with one or more objectives. The MCDM technique may involve a high number of choice alternatives. Solving a MCDM problem entails selection. Object classes seen in the actual physical world rarely have perfectly defined membership criteria. For instance, it goes without saying that dogs, horses, and birds are included in the animal class., and so on, while plainly excluding items such as rocks, fluids, plants, etc. But, other items, like the starfish and microorganisms, possess an equivocal level in terms of animal classification. The similar type of uncertainty emerges when a number like 10 is compared to all actual numbers' "class" which are substantially larger than one. Evidently, "the category of all actual numbers, a substantial larger than one," " both " the level of tall men" and "the class of beautiful women are neither classes nor sets in the way that these terms are commonly used in mathematics. In human cognition, vaguely defined "classes" are important, particularly in abstraction, communication, and pattern recognition. This note's goal is to examine A some of the fundamental elements and ramifications of a notion that could be advantageous in handling "classes" of the kind described over.

The phrase "fuzzy set"[3] describes a "class" that has different membership levels. As will be seen in the sequel, the concept of a fuzzy set provides a convenient beginning the point for the creation of a conceptual structure that is more general and may have a far wider range of applications, especially in the domains of pattern classification and information processing, even if it is comparable to the framework used for ordinary sets. In essence, such a framework provides an organic method for addressing issues when the absence of precisely specified requirements for class membership, instead than the existence of arbitrary variables, is the source of imprecision. Zadeh (1965) illustrated the fuzzy set (FS) to adjust with unclear or ambiguous details while making decisions. The FS consists of a membership function that gives each member a membership value between 0 and 1 target. Atanssov (1986) The intuitionistic fuzzy set (IFS) was proposed, [4,] which is associates each element of a universe with both functions related to membership and those that are not. As a result, it can provide more accurate and definite descriptions than a fuzzy set. But in some real-world situations, the sum number of members and degrees that are not membership to which an item satisfies a quality proposed by an expert could be greater than one, but their one is less than or equal to the square sum. As a result, IFS fails to cope with such a circumstance. As a result, Yager (2013) developed The Pythagorean fuzzy set is a novel fuzzy set,[5] which is named after Pythagoras, a prominent philosopher, thinker, and mathematician. The year was 2017 presented the idea of a q-rung Ortho pair fuzzy set (q-ROFS), which is an extension of the Pythagorean fuzzy set (PFS) and intuitionistic fuzzy set (IFS) [6]. A q-ROFS, like IFS and PFS, has an Ortho pair $y_f(f_x), z_f(f_x)$ of the degree of membership $y_f(f_x)$ and degree of non membership $z_f(f_x)$. In IFS, the Orthopair $y_f(f_x), z_f(f_x)$ bears the limitation $0 \leq \left(y_f(f_x)\right)^q + \left(z_f(f_x)\right)^q \leq 1$. but in PFS, the restriction is $0 \leq \left(y_f(f_x)\right)^q + \left(z_f(f_x)\right)^q \leq 1$. Unlike IFS As required by the q-ROFS and PFS, the orthopair $y_f(f_x), z_f(f_x)$ meet the requirement. $0 \leq \left(y_f(f_x)\right)^q + \left(z_f(f_x)\right)^q \leq 1$for q > 1. Of course, a q-ROFS degenerates into a PFS for $q = 2$, an IFS for $q = 1$, and as q rises, so does the universe of admissible orthopairs. As a result, the q-ROFS contains more acceptable orthopairs than the IFS and PFS, allowing those to express their opinions more freely through an orthopair of membership and non-membership degrees. Preference relations (PRs) are an effective and widely used approach for modelling Issues with decision-making, in which Information on decision makers' preferences across options through comparison between two people. Numerous kinds of preference relations have been put forth and studied using various representations of preference information, including multiplicative preference linguistic preferred relations [9,10], intuitionistic fuzzy preferred relations (IFPRs) [11,12], Pythagorean fuzzy preferred relations (PFPRs) [13,14], fuzzy preferred relations (FPRs) [7,8], and preference relations, where in q-ROFS represents preference information as an orthopair, have not yet been studied. We will concentrate on this issue in this work and propose some novel q-ROFS-based preference relations.

The real world is filled with unreliability, inaccuracy, and ambiguity. In fact, the majority of the notions we encounter in ordinary life are more ambiguous than exact. A lot of writers have recently expressed an interest in modelling ambiguity conventional instruments aren't always effective at solving these difficulties. While a variety many current ideas, including fuzzy set theory and probability theory [15], theory of rough sets [16], ambiguous set theory [17], as well as interval mathematics is well recognized and often-used mathematical methods for modelling ambiguity. In 1999, Molodtsov developed soft set theory as a fresh approach to mathematics for addressing uncertainty, which avoids the issues that affect current techniques [18]. The fuzzy soft set that is trapezoidal notion was proposed by merging the fuzzy trapezoidal number with the soft set. A fuzzy set that is more general called the multi-fuzzy set was recently constructed by authors [19.20], built from ordinary fuzzy sets with an ordered series of conventional fuzzy membership operations. The concept of many fuzzy sets offers a novel way to express A few issues that are hard to describe in existing theory of fuzzy sets extensions, such the color of points. The goal regarding this study is to merge the multi-fuzzy set with the soft set, resulting in a fresh soft set model known as the multi-fuzzy soft set. We can get a fresh soft set model known as the hyperbolic soft fuzzy set. To begin our conversation, we will first go over Fuzzy, fluffy settings that are soft, and multi-fuzzy sets [21,22]. The researchers are interested in q-rung Ortho pair fuzzy soft rough sets [23,24], a hybrid intelligent structure combining soft sets [25], rough fuzzy soft sets [26], and q-rung Ortho pair fuzzy soft sets [27,28]. These sets are a useful mathematical tool for dealing with indeterminate, inconsistent, and incomplete data. The investigation reveals that the role of aggregation operators a critical part of the decision-making process by aggregating the collectively evaluated information from several sources into a single value. As far as we know, no one has talked about using aggregation operators with the pairing of q-ROFS, soft set, and rough set in the q-ROF environment.

When all of this research is analyzed, it is clear that only fuzzy circle and fuzzy parabola curves are investigated from the conics. Fuzzy systems are utilized to plan technological structures that are evolving in the field of engineering today. Hyperbola curves, on the other hand, are widely used in radar wave distribution, point matching in scanning machines, and even plant carbon dioxide distribution. While our goal was to investigate how the fuzzy hyperbola could be mathematically defined, calculated, and graphed, we thought it would be beneficial to concentrate on integrating the applications listed above. We explore and prove these calculations by evaluating past investigations.

Although fuzzy set theory and its extensions have been widely used to handle uncertainty in decision-making problems, existing models still face limitations in representing complex and nonlinear uncertainty. In particular, traditional fuzzy sets and their variants do not adequately capture hyperbolic behavior in uncertain environments. While hyperbolic fuzzy sets have been introduced to address this issue, their application remains limited and lacks integration with parameterized frameworks such as soft set theory. Moreover, there is a lack of comprehensive decision-making models that combine hyperbolic fuzzy sets with soft sets to effectively handle both uncertainty and parameterization simultaneously. Existing multi-criteria decision-making approaches also do not fully explore this combination, resulting in reduced flexibility and accuracy in practical applications. Therefore, this study aims to fill this gap by developing a hyperbolic fuzzy soft set framework and integrating it with an advanced decision-making method, providing a more robust, flexible, and accurate approach for solving complex real-world problems.

Novelties: The main novelties of this study are summarized as follows:

i. This study proposes the concept of the hyperbolic fuzzy soft set, which extends traditional fuzzy sets and soft set theory to better capture complex and nonlinear uncertainty.

ii. The proposed framework incorporates hyperbolic membership functions, providing greater flexibility and improved representation of real-world vague and imprecise information.

iii. Novel aggregation operators based on Einstein t-norms and t-conorms are defined under the hyperbolic fuzzy soft environment to enhance information fusion.

iv. The EDAS method is extended and adapted to the hyperbolic fuzzy soft set framework, enabling more reliable multi-criteria decision-making.

v. The proposed model enhances robustness and accuracy compared to existing fuzzy and soft set-based decision models.

vi. The effectiveness of the approach is demonstrated through a real-world case study (robot selection), showing its suitability for solving complex decision-making problems.

Motivation: In recent years, fuzzy set theory has remained a crucial instrument for handling uncertainty and imprecision in real-world problems. Among its extensions, hyperbolic fuzzy sets give a versatile framework by combining degrees of membership and non-membership with hyperbolic representation, resulting in a more complex structure for uncertainty modelling. However, hyperbolic fuzzy sets alone may not be sufficient to convey the complexities of parameterized decision-making systems. To solve this constraint, the concept of soft sets has gained widespread acceptance for its ability to accommodate parameterized uncertainty without requiring sophisticated membership functions.

The goal of this research is to construct and formalize the hypothesis of hyperbolic fuzzy soft sets by converting and expanding on existing hyperbolic fuzzy set theory findings.

i. To develop a new decision-making model base on the hyperbolic fuzzy soft set.
ii. To develop a new aggregation t-norm and t-conorm-based operator proposed by Einstein for the proposed concept.
iii. To modify the EDAS under the hyperbolic fuzzy soft set.
iv. To apply this making- decisions model in the real-world issues

Contribution: To make sure to choose the optimal robot, we present a new decision-making model in this paper that is according to the EDAS approach under the hyperbolic fuzzy soft set. The following lists this article's primary contributions:
i. We presented the idea of hyperbolic fuzzy soft set.
ii. We specify what the aggregate is operator based on Einstein t-norms and t-conorm for the proposed concept.
iii. We modify the EDAS method for hyperbolic soft set.
iv. We apply the proposed model to select robot selection.
v. To assess the suggested model's precision and effectiveness, we contrasted it with the current MCDM model.

Table 1: The current study makes use of the given acronyms.

MCDM	Multi criteria decision making
IFS	Fuzzy intuitionistic sets
PFSs	Pythagorean fuzzy sets
GIFSS	Generalized intuitionistic fuzzy soft set
HYFS	Hyperbolic Fuzzy Set
HFSS	Hyperbolic Fuzzy Soft Set
MD	Membership degree
NMD	Non membership degree
OD	Optimism degree
PD	pessimism degree

2. Preliminaries

We establish the groundwork in this section by going over key ideas and background data. We start with pertinent definitions and characteristics that have helped to guide our investigation. By giving a thorough summary of this foundational information, we prepare the ground for the introduction of novel approaches later on in the research.

Definition 1: We take into $\tilde{\mathcal{R}} = \{\langle f_x : x = 1,2, \dots \dots . n \rangle\}$ as a worldwide collection. The following is an expression for a fuzzy set B:

$$B = \{\langle f_x , y_B(f_x)\rangle; f_x \in \tilde{\mathcal{R}}\}, \tag{1}$$

where the function is $y_B(f_x): \mathcal{R} \longrightarrow [0,1]$ represent the MD.

Definition 2: Let $B_1 = \{\langle f_x , y_{B_1}(f_x)\rangle; f_x \in \tilde{\mathcal{R}}\}$ and $B_2 = \{\langle f_x , y_{B_2}(f_x)\rangle; f_x \in \tilde{\mathcal{R}}\}$ be the two fuzzy set. We define the following types of set operations on FS$(\tilde{\mathcal{R}})$:

i. $B_1 \subseteq B_2 \ iff \ y_{B_1}(f_x) \leq y_{B_2}(f_x)$.
ii. $B_1^C = \langle 1 - y_{B_1}(f_x)\rangle$.
iii. $B_1 \cup B_2 = \langle max(y_{B_1}(f_x), y_{B_2}(f_x))\rangle$.
iv. $B_1 \cap B_2 = \langle min(y_{B_1}(f_x), y_{B_2}(f_x))\rangle$.
v. $B_1 + B_2 = \langle (y_{B_1}(f_x) + y_{B_2}(f_x) - y_{B_1}(f_x) y_{B_2}(f_x))\rangle$.
vi. $B_1 \times B_2 = \langle (y_{B_1}(f_x) \times y_{B_2}(f_x))\rangle$.

Definition 3: Let $\tilde{\mathcal{R}}$ be defined as a universal collection. An IFS $\mathscr{b}$ is defined as

$$\mathscr{b} = \{\langle f_x , y_{\mathfrak{s}}(f_x), z_{\mathfrak{t}}(f_x)\rangle; f_x \in \tilde{\mathcal{R}}\}, \tag{2}$$

where the functionalities are located $y_{\mathfrak{s}}(f_x) : \tilde{\mathcal{R}} \rightarrow [0,1]$ and $z_{\mathfrak{t}}(f_x) : \tilde{\mathcal{R}} \rightarrow [0,1]$ defne the MD and ND in such a way that $0 \leq (y_{\mathfrak{s}}(f_x)) + (z_{\mathfrak{t}}(f_x)) \leq 1$.

Supposing $(y_{\mathfrak{s}}(f_x)) + (z_{\mathfrak{t}}(f_x)) \leq 1$, then MD is define by HA$(f_x) = 1 - [(y_{\mathfrak{s}}(f_x)) + (z_{\mathfrak{t}}(f_x))]$ and HA$(f_x) \in [0,1]$ such that $(y_{\mathfrak{s}}(f_x)) + (z_{\mathfrak{t}}(f_x)) + (HA(f_x)) = 1$.

Definition 4: We treat $\tilde{\mathcal{R}}$ as a Complete collection. A PFS $\mathfrak{z}$ is defined as

$$\mathfrak{z} = \{\langle f_x, y_3(f_x), z_{\mathfrak{a}}(f_x)\rangle; f_x \in \tilde{\mathcal{R}}\}, \tag{3}$$

where the function $y_3(f_x) : \tilde{\mathcal{R}} \rightarrow [0,1]$ and $z_{\mathfrak{a}}(f_x) : \tilde{\mathcal{R}} \rightarrow [0,1]$ defne The ND and MD in such a way that supposing $0 \leq (y_3(f_x))^2 + (z_{\mathfrak{a}}(f_x))^2 \leq 1$.

Supposing $(y_3(f_x))^2 + (z_{\mathfrak{a}}(f_x))^2 \leq 1$. Then MD is defined by $H_A = 1 - \left[(y_3(f_x))^2 + (z_{\mathfrak{a}}(f_x))^2\right]^{1/2}$. and $H_A(f_x) \in [0,1]$ such that $(y_3(f_x))^2 + (z_{\mathfrak{a}}(f_x))^2 + (H_A(f_x))^2 = 1$.

Definition 5: We take into $\mathcal{R}$ as a universal collection. The q-ROFS $\mathfrak{U}$ is expressed as

$$\mathfrak{U} = \{\langle f_x, y_A(f_x), z_B(f_x)\rangle; f_x \in \mathcal{R}\}, \tag{4}$$

where the functions are $y_{\mathfrak{T}}(f_x) : \mathcal{R} \rightarrow [0,1]$ and $z_f(f_x) : \mathcal{R} \rightarrow [0,1]$ defne the MD and ND such a way that $0 \leq (y_{\mathfrak{T}}(f_x))^q + (z_{\mathfrak{R}}(f_x))^q \leq 1$.

Supposing $(y_{\mathfrak{T}}(f_x))^q + (z_{\mathfrak{R}}(f_x))^q \leq 1$, then HD is defined by $H_{\mathfrak{J}}(f_x) = 1 - [y_{\mathfrak{T}}(f_x)^q + z_{\mathfrak{R}}(f_x)^q]^{1/q}$ and $H_{\mathfrak{J}}(f_x) \in [0,1]$such that $(y_{\mathfrak{T}}(f_x))^q + (z_{\mathfrak{R}}(f_x))^q + (H_{\mathfrak{J}}(f_x))^q = 1$.

Definition 6: Take into consideration a set of q-ROFSs represented as $F_\zeta = (y_\zeta, z_\zeta)$ for $\zeta = 1,2,\ldots,n$. Let q − ROFWA : $G^n \rightarrow G$ be a mapping that is described as

$$\text{q} - \text{ROFWA } (F_1, F_2, \ldots, F_n) = \langle\left(1 - \prod_{\zeta=1}^{n}(1 - y_\zeta^q)^{\frac{1}{q}}, \prod_{\zeta=1}^{n} z_\zeta^{v\zeta}\right)\rangle \tag{5}$$

where G represent the collection of every q-ROFS, and $\varsigma = (\varsigma_1, \varsigma_2, \ldots . \varsigma_n)$ is connected to a weight vector with $(F_1, F_2, \ldots, F_n)$ .The weights vk satisfy $0 \leq v_\zeta \leq 1$ and $\sum_{\zeta=1}^{n} v_\zeta = 1$. The mapping the weighted averaging operator for q-rung orthopairs, or q − ROFWA, is a member of the set of q-ROFSs.

Definition 7: Take a look at a collection of $M_\wp = (y_\wp, z_\wp)$ for $\wp = 1,2,\ldots,n$.

Assume that the mapping q − ROFWG : $G^n \rightarrow G$ is defined as

$$\text{q–ROFWG } (F_1, F_2, \ldots \ldots F_n) = \langle\prod_{\wp=1}^{n} y_{\mathfrak{X}}\rangle \langle\prod_{\wp=1}^{n} y_{\mathfrak{X}}^{v\wp}, \left(1 - \prod_{\wp}^{n}(1 - Z_{\mathfrak{Y}})^{v\wp}\right)^{1/q}\rangle, \tag{6}$$

where G is the collection of all q-ROFSs and $\mathrm{v} = (\mathrm{v}_1, \mathrm{v}_2 \ldots \ldots, \mathrm{v}_\mathrm{n})$ vector associated with $\wp = (\mathrm{F}_1, \mathrm{F}_2 \ldots \ldots, \mathrm{F}_\mathrm{n})$.The weights $v_\wp$ fulfil $0 \leq \mathrm{v}_\wp \leq 1$ and $\sum_{\wp=1}^{\mathrm{n}} \mathrm{v}_\wp = 1$. The mapping q-rung orthopair weighted geometric operator, or q − ROFWG, is a member of the set of q-ROFSs.

Definition 8: $\mathcal{R}$ is regarded as a universal set. One way to express a HyFS ξ is as

$$\xi = \{\langle f_x, y_\xi(f_x), z_\xi(f_x)\rangle : f_x \epsilon\, \tilde{\mathcal{R}}\}, \tag{7}$$

where $y_\xi(f_x)$: $\tilde{\mathcal{R}} \rightarrow [0,1]$ and $z_\xi(f_x) : \tilde{\mathcal{R}} \rightarrow [0,1]$ is the degree of pessimism (PD) and optimism (OD), correspondingly, so that $0 \leq (y_\xi(f_x) \times z_\xi(f_x)) \leq 1$.

HyFS$(\tilde{\mathcal{R}})$ is the compilation of all HyFSs over $\tilde{\mathcal{R}}$.

Definition 9: Let $\hat{\hat{\mathrm{U}}}, \ddot{\mathrm{U}} \in$ HyFS $\tilde{\mathcal{R}}$ be two HyFSs, and We see HyFS $\tilde{\mathcal{R}}$ as the entire family. HyFS in $\tilde{\mathcal{R}}$, as provided by $\hat{\hat{\mathrm{U}}} = \langle y_{\hat{\hat{\mathrm{U}}}}(f_x), z_{\hat{\hat{\mathrm{U}}}}(f_x)\rangle$ and $\mathbb{N} = \langle y_{\mathbb{N}}(f_x), z_{\mathbb{N}}(f_x)\rangle$

Next, the set operations listed below are defined on HyFS $(\mathcal{R})$:

i. $\hat{\hat{\mathrm{U}}} \subseteq \ddot{\mathrm{U}}\ iff\ y_{\hat{\hat{\mathrm{U}}}}(f_x) \leq y_{\ddot{\mathrm{U}}}(f_x)$ and $z_{\hat{\hat{\mathrm{U}}}}(f_x) \geq z_{\ddot{\mathrm{U}}}(f_x)$.

ii. $\hat{\hat{\mathrm{U}}}^c = \langle 1 - y_{\hat{\hat{\mathrm{U}}}}(f_x), 1 - z_{\ddot{\mathrm{U}}}(f_x)\rangle$.

iii. $\hat{\hat{\mathrm{U}}} \cup \ddot{\mathrm{U}} = \langle max\left(y_{\hat{\hat{\mathrm{U}}}}(f_x), y_{\hat{\hat{\mathrm{U}}}}(f_x), min(z_{\ddot{\mathrm{U}}}(f_x), z_{\ddot{\mathrm{U}}})\right)\rangle$.

iv. $\hat{\hat{\mathrm{U}}} \cap \ddot{\mathrm{U}} = \langle min\left(y_{\hat{\hat{\mathrm{U}}}}(f_x), y_{\hat{\hat{\mathrm{U}}}}(f_x)\right), max(z_{\ddot{\mathrm{U}}}(f_x), z_{\ddot{\mathrm{U}}}(f_x))\rangle$.

v. $\hat{\hat{\mathrm{U}}} + \ddot{\mathrm{U}} = \langle\left(y_{\hat{\hat{\mathrm{U}}}}(f_x) + y_{\ddot{\mathrm{U}}}(f_x) - y_{\hat{\hat{\mathrm{U}}}}(f_x) y_{\hat{\hat{\mathrm{U}}}}(f_x), (z_{\hat{\hat{\mathrm{U}}}}(f_x) z_{\ddot{\mathrm{U}}})\right)\rangle$.

vi. $\hat{\hat{\mathrm{U}}} \times \ddot{\mathrm{U}} = \langle\left(y_{\hat{\hat{\mathrm{U}}}}(f_x) y_{\ddot{\mathrm{U}}}(f_x)\right), \left(z_{\hat{\hat{\mathrm{U}}}}(f_x) + z_{\ddot{\mathrm{U}}}(f_x) - z_{\hat{\hat{\mathrm{U}}}}(f_x) z_{\ddot{\mathrm{U}}}(f_x)\right)\rangle$.

Definition 10: Let X be a universal set, and B be a collection of attributes. A soft set over X is a pair (β, E), with $E \subseteq B$ and $\beta: E \rightarrow \mathcal{P}(X)$. $\mathcal{P}(X)$ represents the power set of X.

A soft set can be mathematically represented as follows:

$$(\beta, E) = \{(\gamma, \beta(\gamma)) \mid \gamma \in E, \beta(\gamma) \in \mathcal{P}(X)\} \tag{8}$$

Each $\gamma \in E$ is called a parameter, and $\beta(\gamma)$ is the set of elements in X associated with γ.

Definition 11: The union of two soft sets $(\hat{\mathrm{I}}, \hat{\hat{\mathrm{W}}})$ and $(\tilde{\mathcal{F}}, \ddot{\mathcal{G}})$ over a common universe U, written as $(\hat{\mathrm{I}}, \hat{\hat{\mathrm{W}}})\ \tilde{\cup}\ (\tilde{\mathcal{F}}, \ddot{\mathcal{G}})$, is the soft set $(\hat{\mathrm{O}}, \ddot{\mathrm{O}})$, where $\ddot{\mathrm{O}} = \hat{\hat{\mathrm{W}}} U \ddot{\mathcal{G}}$ and $e \in \ddot{\mathrm{O}}$.

i. $(\hat{\mathrm{I}}, \hat{\hat{\mathrm{W}}})\ \tilde{\cup}\ (\tilde{\mathcal{F}}, \ddot{\mathcal{G}})\ \tilde{\cup}\ (\hat{\mathrm{O}}, \ddot{\mathrm{O}}) = (\hat{\mathrm{I}}, \hat{\hat{\mathrm{W}}})\ \tilde{\cup}\ (\tilde{\mathcal{F}}, \ddot{\mathcal{G}})\ \tilde{\cup}\ (\hat{\mathrm{O}}, \ddot{\mathrm{O}})$.

ii. $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\,\mho_{\widehat{U}}=\mho_{\widehat{U}}\,,\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\,\mho_{E}=\mho_{E}\,\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\,\emptyset_{\widehat{U}}=\left(\hat{\hat{I}},\widehat{U}\right).$

iii. $\left(\hat{\hat{I}},\widehat{U}\right)$ need not be a soft set subset of $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$ But if $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\subseteq}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$Then $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\subseteq}\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$ more over $\left(\hat{\hat{I}},\widehat{U}\right)=\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$.

iv. $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\widehat{U}\right)=\emptyset_{\widehat{U}}\Leftrightarrow\left(\hat{\hat{I}},\widehat{U}\right)\emptyset_{\widehat{U}}$ and $\left(\tilde{\mathcal{F}},\widehat{U}\right)=\emptyset_{\widehat{U}}$.

v. $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\Cap(\hat{O},\ddot{O})=\left(\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\right)\Cap\left(\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\right)\Cap\left(\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\,(\hat{O},C)\right).$

vi. $\left(\left(\hat{\hat{I}},\widehat{U}\right)\Cap\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\right)\tilde{\cup}\,(\hat{O},\ddot{O})=\left(\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\cup}\,(\hat{O},\ddot{O})\right)\Cap\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\tilde{\cup}\,(\hat{O},\ddot{O}).$

Definition 12: The restricted union of two soft sets $(\hat{\hat{I}},\widehat{U})$ and $(\tilde{\mathcal{F}},\ddot{\oint})$ over a universe U, written as $(\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\tilde{\mathcal{F}},\ddot{\oint})\,R$, is the soft set universe U, $(\hat{O},\ddot{O})$, where $\ddot{O}=\widehat{U}\Cap\ddot{\oint}\neq\emptyset$ and $e\in\ddot{O}$.

i. $\left(\hat{\hat{I}},\widehat{U}\right)\cup_{\mathcal{R}}((\tilde{\mathcal{F}},\ddot{\oint})\cup_{\mathcal{R}}(H,C))=((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\ddot{\oint},\ddot{\oint}))\cup_{\mathcal{R}}(\hat{O},\ddot{O}).$

ii. $\left(\hat{\hat{I}},\widehat{U}\right)\cup_{\mathcal{R}}\mho_{\widehat{U}}=\mho_{\widehat{U}},\left(\hat{\hat{I}},\widehat{U}\right)\cup_{\mathcal{R}}\mho_{E}=\mho_{\widehat{U}}\,,(\hat{\hat{I}},\ddot{E})\cup_{\mathcal{R}}\emptyset=\left(\hat{\hat{I}},\widehat{U}\right),\left(\hat{\hat{I}},\widehat{U}\right)\cup_{\mathcal{R}}\emptyset_{E}=\left(\hat{\hat{I}},\widehat{U}\right).$

iii. $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\not\subseteq}\left(\hat{\hat{I}},\widehat{U}\right)\cup_{\mathcal{R}}\left(\ddot{\oint},\ddot{\oint}\right)$, in general. But if $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\not\subseteq}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$, then $(\hat{\hat{I}},\ddot{E})\tilde{\not\subseteq}(\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\ddot{\oint},\ddot{\oint})$, moreover $(\hat{\hat{I}},\widehat{U})=(\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\tilde{\mathcal{F}},\ddot{\oint})$.

iv. $\left(\hat{\hat{I}},\widehat{U}\right)\cup_{\mathcal{R}}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)=\emptyset_{\widehat{U}}\Leftrightarrow\left(\hat{\hat{I}},\widehat{U}\right)=\emptyset_{\widehat{U}}$ and $\left(\tilde{\mathcal{F}},\widehat{U}\right)=\emptyset_{\widehat{U}}$.

v. $(\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}((\tilde{\mathcal{F}},\ddot{\oint})\Cap(\hat{O},\ddot{O}))=((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\tilde{\mathcal{F}},\ddot{\oint}))\Cap((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\hat{O},\ddot{O})).$

vi. $((\hat{\hat{I}},\widehat{U})\Cap(\tilde{\mathcal{F}},\ddot{\oint}))\cup_{\mathcal{R}}(\hat{O},\ddot{O})=((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\hat{O},\ddot{O}))\Cap((\tilde{\mathcal{F}},\ddot{\oint})\cup_{\mathcal{R}}(\hat{O},\ddot{O})).$

vii. $(\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}((\tilde{\mathcal{F}},\ddot{\oint})\sqcap_{\epsilon}(\hat{O},\ddot{O}))=((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\tilde{\mathcal{F}},\ddot{\oint}))\sqcap_{\epsilon}((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\hat{O},\ddot{O})).$

Definition 13: The extended intersection of two soft sets $(\hat{\hat{I}},\widehat{U})$ and $(\tilde{\mathcal{F}},\ddot{\oint})$ over a common universe U, termed $(\hat{\hat{I}},\widehat{U})\sqcap_{\epsilon}(\tilde{\mathcal{F}},\ddot{\oint})$, is the soft set $(\hat{O},\ddot{O})$, where $C=\hat{\hat{I}}\sqcap_{\epsilon}\ddot{\oint}$ and $e\in\ddot{O}$.

i. $\left(\hat{\hat{I}},\widehat{U}\right)\sqcap_{\epsilon}\left(\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\sqcap_{\epsilon}(\hat{O},\ddot{O})\right)=\left(\left(\hat{\hat{I}},\widehat{U}\right)\sqcap_{\epsilon}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\right)\sqcap_{\epsilon}(\hat{O},\ddot{O}).$

ii. $\left(\hat{\hat{I}},\widehat{U}\right)\sqcap_{\epsilon}\mho_{\widehat{U}}=(\hat{\hat{I}},\widehat{U}),(\hat{\hat{I}},\widehat{U})\sqcap_{\epsilon}\emptyset_{\widehat{U}}=\emptyset_{\widehat{U}}$.

iii. $\left(\hat{\hat{I}},\widehat{U}\right)\sqcap_{\epsilon}\left(\ddot{\oint},\ddot{\oint}\right)\tilde{\not\subseteq}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$, in general. But if $\left(\hat{\hat{I}},\widehat{U}\right)\tilde{\not\subseteq}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$, then $\left(\hat{\hat{I}},\widehat{U}\right)\sqcap_{\epsilon}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)\tilde{\not\subseteq}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)$, moreover $\left(\hat{\hat{I}},\widehat{U}\right)\sqcap_{\epsilon}\left(\tilde{\mathcal{F}},\ddot{\oint}\right)=(\tilde{\mathcal{F}},\ddot{\oint})$.

iv. $(\hat{\hat{I}},\widehat{U})\sqcap_{\epsilon}((\tilde{\mathcal{F}},\ddot{\oint})\cup_{\mathcal{R}}(\hat{O},\ddot{O}))=((\hat{\hat{I}},\widehat{U})\sqcap_{\epsilon}(\tilde{\mathcal{F}},\ddot{\oint}))\cup_{\mathcal{R}}((\hat{\hat{I}},\widehat{U})\sqcap_{\epsilon}(\hat{O},\ddot{O})).$

v. $((\hat{\hat{I}},\widehat{U})\cup_{\mathcal{R}}(\tilde{\mathcal{F}},\ddot{\oint}))\sqcap_{\epsilon}(\hat{O},\ddot{O})=((\hat{\hat{I}},\widehat{U})\sqcap_{\epsilon}(\hat{O},\ddot{O}))\cup_{\mathcal{R}}((\tilde{\mathcal{F}},\ddot{\oint})\sqcap_{\epsilon}(\hat{O},\ddot{O})).$

3. *Hyperbolic Fuzzy Soft Set*

In this section we provide a more detailed and coherent descriptive analysis of the hyperbolic fuzzy soft set. We have refined the definitions, elaborated the underlying mathematical structure, and improved the explanation of its properties to ensure better understanding. The relationship between the proposed model and existing approaches is now more clearly articulated, highlighting its advantages and applicability.

Definition 14: Let $X\neq\emptyset$ be the universe of discourse E be the set of parameters. Then the hyperbolic fuzzy soft set defined as

$$(F,A)=\{e_i,F(e_i)|e_i\in F,F(e_i)\in P(A)\subseteq X\}. \tag{9}$$

$$F:E\rightarrow HFSS(X).\ \ F(e_i)=\{\langle x:\mu_x,\gamma_x\rangle\ |x\in X,e_i\in E\}.$$

Where μ_x (MD) member ship degree and γ_x (NMD) non member ship degree.

Definition 15: Let (F,A) and (G,B) be the hyperbolic fuzzy soft Set $X\neq\emptyset$ be the universe of discourse, where $F,G:E\rightarrow HFSS(X)$ and for each parameter e_i

$$F(e_i)=\{\langle x:\mu^F{}_x,\gamma^F{}_x|x\in X\rangle\} \tag{10}$$

$$G(e_i)=\{\langle x:\mu^G{}_x,\gamma^G{}_x|x\in X\rangle\} \tag{11}$$

Then the basic properties for (F,A) and (G,B) are define follows:

1) $(F,A)=(G,B)\Leftrightarrow A=B\ and\ \forall e_i\in A,F(e_i)=G(e_i).$
2) $F^c(e_i)=\{\langle x:\gamma^F{}_x,\mu^F{}_x\rangle x\in X\}.$
3) $\mu^H{}_x=max(\mu^F{}_x,\mu^G{}_x),\gamma^H{}_x=min(\gamma^F{}_x,\gamma^G{}_x).$
4) $\mu^H{}_x=min(\mu^F{}_x,\mu^G{}_x),\gamma^H{}_x=max(\gamma^F{}_x,\gamma^G{}_x).$

5) $\mu^F{}_x \leq \mu^G{}_x, \gamma^F{}_x \geq \gamma^G{}_x, \forall \gamma^F{}_x$

Definition 16: Let a be hyperbolic fuzzy soft Set then the Score function as,
$\mathbb{S}_{pe} = \frac{1+(\Xi)^2-(\Lambda)^2}{2}$,such that $\mathbb{S}_{pe} \in [0,1]$.

Definition 17: The following are suitable substitutes for algebraic t-norm and t-conorm, respectively: Einstein sum $\oplus\, \varepsilon$ and Einstein product $\otimes\, \varepsilon$:

$$\eta \oplus \mu = \frac{\eta+\mu}{1+\eta.\mu}. \tag{12}$$

$$\eta \otimes \mu = \frac{\eta+\mu}{1+(1-\eta).(1-\mu)} \qquad \forall(\eta,\mu) \in [0,1]^2. \tag{13}$$

An Einstein product is called a t-norm. $\otimes\, \varepsilon$, whereas the Einstein sum is a t-conorm. $\oplus\, \varepsilon$,
The t-norm $(\alpha \oplus_\varepsilon \beta)$ and t-conorm $(\alpha \otimes_\varepsilon \beta)$ satisfy boundary, monotonicity, commutativity, and associativity properties, respectively.

Definition 18: Let $ℸ_1 = (\mu_1, \gamma_1)$ and $ℸ_2 = (\mu_2, \gamma_2)$be the two hyperbolic fuzzy soft set and $\alpha > 0$ then we have the Einstein based operations as follows:

1. $ℸ_1 \oplus_\varepsilon ℸ_2 = \left(\frac{\mu_1+\mu_2}{1+\mu_1.\mu_2}, \frac{\gamma_1.\gamma_2}{1+(1-\gamma_1).(1-\gamma_2)}\right)$.
2. $ℸ_1 \otimes \varepsilon ℸ_2 = \left(\frac{\mu_1.\mu_2}{1+(1-\mu_1).(1-\mu_2)}, \frac{\gamma_1+\gamma_2}{1+\gamma_1.\gamma_2}\right)$.
3. $\alpha_\epsilon ℸ = \frac{(1+\mu)^\alpha-(1-\mu)^\alpha}{(1+\mu)^\alpha+(1-\mu)^\alpha}, \frac{2(\gamma)^\alpha}{(2-\gamma)^\alpha+(\gamma)^\alpha}$.
4. $ℸ^\alpha = \frac{2(\mu)^\alpha}{(2-\mu)^\alpha+(\mu)^\alpha}, \frac{(1+\gamma)^\alpha-(1-\gamma)^\alpha}{(1+\gamma)^\alpha+(1-\gamma)^\alpha}$.

Definition 19: Let $\beta_k = \langle \mathcal{U}_{\beta k}, \mathcal{V}_{\beta k} \rangle$ $(k = 1,2,\ldots,n)$be a collection of HFFSS and $\alpha = \{\alpha_1, \alpha_k, \alpha_k, \ldots, \alpha_k\}$ be the wights with condition that $\alpha_k > 0$ and $\sum_{k=1}^n \alpha_k = 1$. Then the HFSEWAAAO is mapping: $\beta^n \rightarrow \beta$.

$$\text{HFSEWAO}\ (\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n) = \left(\frac{\prod_{k=1}^n (1+\mathcal{U}_{ki})^{\alpha_k} - (\prod_{k=1}^n (1-\mathcal{U}_{ki})^{\alpha_k})}{(\prod_{k=1}^n (1+\mathcal{U}_{ki})^{\alpha_k}) + (\prod_{k=1}^n (1-\mathcal{U}_{ki})^{\alpha_k})}, \frac{2(\prod_{k=1}^n (\mathcal{V}_{ki})^{\alpha_k})}{(\prod_{k=1}^n (2-\mathcal{V}_{ki})^{\alpha_k}) + (\prod_{k=1}^n (\mathcal{V}_{ki})^{\alpha_k})}\right) \tag{14}$$

Theorem1: Let $\beta_k = \langle \mathcal{U}_{\beta k}, \mathcal{V}_{\beta k} \rangle$ $(k = 1,2,\ldots,n)$be a collection of HFFSS and $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\}$ be the wights with condition that $\alpha_k > 0$ and $\sum_{k=1}^n \alpha_k = 1$.
The total aggregate obtained be making use of the HFSEWAO operator is also an HFSNs.

$$\text{HFSEWAO}\ (\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n) = \left(\frac{\prod_{k=1}^n (1+\mathcal{U}_{ki})^{\alpha_k} - (\prod_{k=1}^n (1-\mathcal{U}_{ki})^{\alpha_k})}{(\prod_{k=1}^n (1+\mathcal{U}_{ki})^{\alpha_k}) + (\prod_{k=1}^n (1-\mathcal{U}_{ki})^{\alpha_k})}, \frac{2(\prod_{k=1}^n (\mathcal{V}_{ki})^{\alpha_k})}{(\prod_{k=1}^n (2-\mathcal{V}_{ki})^{\alpha_k}) + (\prod_{k=1}^n (\mathcal{V}_{ki})^{\alpha_k})}\right). \tag{15}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^U$ U is the weight vector of $\beta_k (j = 1,2,\ldots,n)$ such that $\alpha_k \in [0,1], k = 1,2,\ldots,n$ and $\sum_{k=1}^n \alpha_k = 1$.

Theorem 2: Let $\beta_k = \langle \mathcal{U}_{\beta k}, \mathcal{V}_{\beta k} \rangle (j = 1,2,\ldots,n)$ be an assortment of HFSNs with $\leq \alpha_k$.
$\text{HFSEWAO}(\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n) \leq (\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n)$.
Where $\alpha = (\alpha_1, \alpha_2, \alpha_3 \ldots \ldots \alpha_n)^U$ is the vector of weights of $\beta_k (k = 1,2, \ldots \ldots n)$ such that $\alpha_k \in [0,1], k = 1,2, \ldots \ldots n$ and $\sum_{k=1}^n \alpha_k = 1$.

Theorem 3: Let $\beta_k = \langle \mathcal{U}_{\beta k}, \mathcal{V}_{\beta k} \rangle (k = 1,2,\ldots,n)$ be the collection of HFSNs $\leq \alpha_k$ and $\alpha = (\alpha_1, \alpha_2, \alpha_3 \ldots \ldots \alpha_n)^U$ is the vector of weights of $\beta_k (j = 1,2,\ldots,n)$ such that $\alpha_k \in [0,1], k = 1,2, \ldots \ldots n$ and $\sum_{k=1}^n \alpha_k = 1$. Then

i. (Idempotency): If all $\beta_k (k = 1,2,\ldots,n)$ are equal $i.e., \beta_k = \beta$ for all $k = 1,2,\ldots,n$ then

$(\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n) = \beta$.

ii. (Boundary):

$$\beta_{min} \leq \text{HFSEWAO}(\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n) \leq \beta_{max}\ for\ over\ \alpha$$

Where $\beta_{min} = min_k\{\beta_k\}\ and\ \beta_{max} = max_k\{\beta_k\}$.

iii. (Monotonicity): Let $\beta_k{}^* = (\text{HFFSOWG})\langle \mathcal{V}_{\beta k}{}^*, \mathcal{U}_{\beta k}{}^* \rangle, (k = 1,2,\ldots,n)$ be the collection of HFSNs $\leq M$. And $\mathcal{U}_{\beta k}{}^* \leq \mathcal{U}_{\beta k}{}^*$ and $\mathcal{V}_{\beta k}{}^* \geq \mathcal{V}_{\beta k}{}^*$ for all k, then.
$\text{HFSEWAO}(\beta_1, \beta_2, \beta_3 \ldots \ldots . . \beta_n) \leq (\beta_1{}^*, \beta_2{}^*, \ldots \ldots \ldots \beta_n{}^*)$ for every α.

4. *Decision making model based on EDAS method*

Various decision making involving multi-attribute decision-making (MCDM) have been studied using the standard competing qualities using the EDAS (Evaluation based on Distance from Average Solution) technique (Kesha-varz Ghorabaee et al., 2015). By determining the average solution (AV) using two distance measurements, this model is able to explain the disparity among all of the alternates and the AV: Examples include Negative Distance from Average

(NDA) and Positive Distance from Average (PDA). The option with the largest PDA values and the smallest PDA numbers is the most beneficial.

We let's say there are n options $\{\tilde{\tilde{B}}_1, \tilde{\tilde{B}}_2, \dots \dots \tilde{\tilde{B}}_n\}$, o attributes $\{\tilde{\tilde{H}}_1, \tilde{\tilde{H}}_2, \dots \dots \tilde{\tilde{H}}_o\}$ and u experts $\{\tilde{\tilde{F}}_1, \tilde{\tilde{F}}_{1_2}, \dots \tilde{\tilde{F}}_{1_r}\}$, let $\{\tilde{\tilde{w}}_1, \tilde{\tilde{w}}_2, \dots \dots \tilde{\tilde{w}}_n\}$ and $\{\tilde{\tilde{\emptyset}}_1, \tilde{\tilde{\emptyset}}_2, \dots \tilde{\tilde{\emptyset}}_n\}$ be The weighted vector of the attribute and the opinion of the expert weighting vector that satisfied $w_i \in [0,1], \emptyset \in [0,1]$ and $\sum_{i=1}^{m} \tilde{\tilde{w}}_i = 1 \sum_{r=1}^{u} \tilde{\tilde{\emptyset}}_r = 1$.

The steps are following below.

Step 1: Create an evaluation matrix $Q = (\zeta_{ij}^r) = i = 1,2, \dots \dots n, j = 1,2, \dots \dots o, r = 1,2 \dots . u$.

$$\begin{array}{cc} & G_1 \quad G_2 \dots \dots \dots G_n \\ Q = (\zeta_{ij}^r)_{m \times o} = & \begin{pmatrix} \zeta_{11}^r & \zeta_{12}^r & \cdots & \zeta_{1n}^r \\ \zeta_{21}^r & \zeta_{22}^r & \cdots & \zeta_{2n}^r \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{m1}^r & \zeta_{m2}^r & \cdots & \zeta_{mo}^r \end{pmatrix} \end{array} \tag{18}$$

Where ζ_{ij}^r denoted the HFSS of alternative A_i on the attribute G_j by expert F^u.

Step 2: take expert criteria weight for expert's table.

Step 3: According to the choice matrix $Q = (\zeta_{ij}^r)_{m \times o}$ and experts weighting vector $\{\emptyset_1, \emptyset_2, \dots . \emptyset_n\}$ We are able to get an all things considered $(\zeta_{ij}^{\prime r})$,to $(\zeta_{ij}^{\prime})$ Using HFSSEWAAO, the computational results can be reported as follows.

$$\text{HFSNs} = HFSWA\,(\beta_1, \beta_2, \beta_3 \dots \dots . \beta_n) = \left\langle \frac{\prod_{k=1}^{n}(1+u_{ki})^{\alpha_k} - (\prod_{k=1}^{n}(1-u_{ki})^{\alpha_k})}{(\prod_{k=1}^{n}(1+u_{ki})^{\alpha_k}) + (\prod_{k=1}^{n}(1-u_{ki})^{\alpha_k})}, \frac{2(\prod_{k=1}^{n}(v_{ki})^{\alpha_k})}{(\prod_{k=1}^{n}(2-v_{ki})^{\alpha_k}) + (\prod_{k=1}^{n}(v_{ki})^{\alpha_k})} \right\rangle. \tag{19}$$

$$Q = (\zeta_{ij}^{\prime})_{m \times o} \begin{pmatrix} \zeta_{11}^{\prime} & \zeta_{12}^{\prime} & \cdots & \zeta_{1n}^{\prime} \\ \zeta_{21}^{\prime} & \zeta_{22}^{\prime} & \cdots & \zeta_{2n}^{\prime} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{m1}^{\prime} & \zeta_{m2}^{\prime} & \cdots & \zeta_{mo}^{\prime} \end{pmatrix}. \tag{20}$$

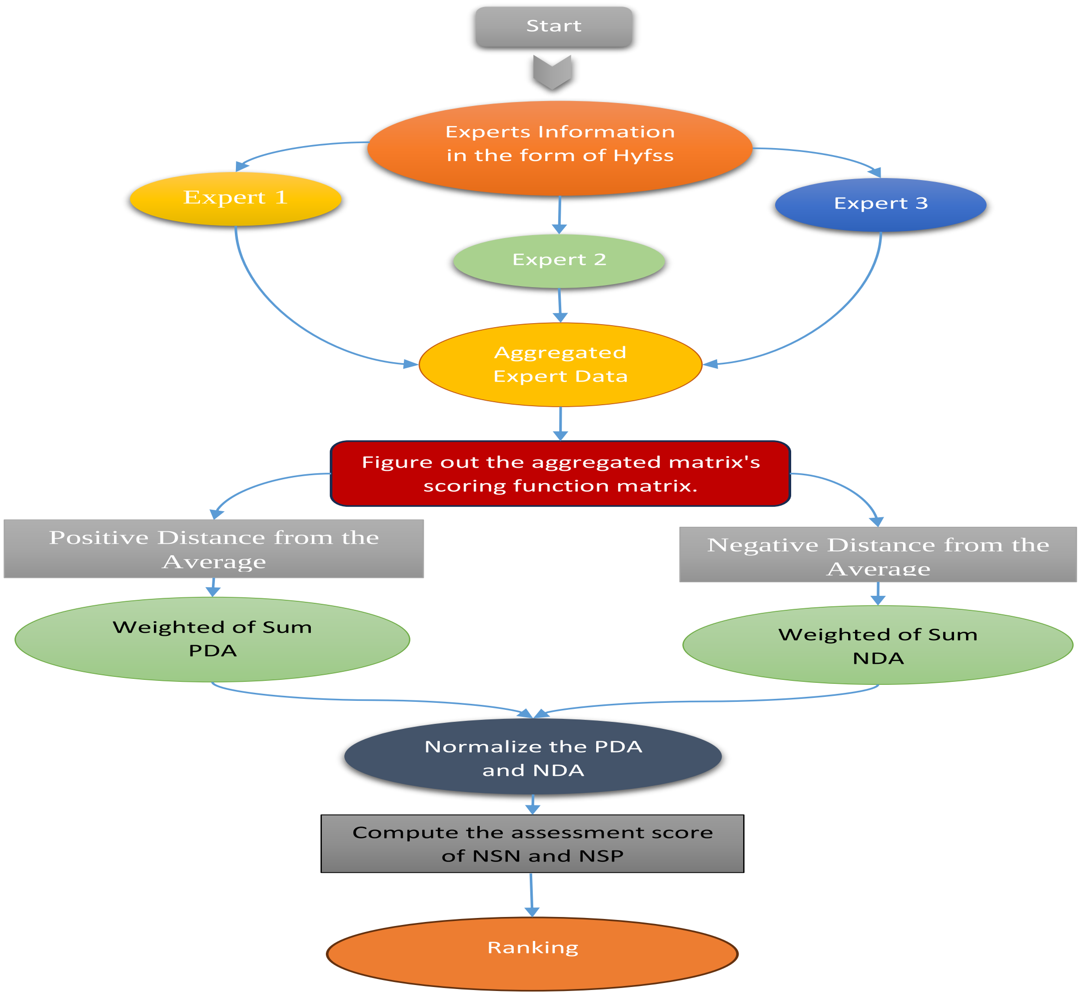

Figure 1: Flowchart of proposed model

Step 4: Calculate the average solution (AV) based on all provided qualities.

$$AV = \left(AV_j\right)_{1\times n} = \left(\frac{\sum_i^n \zeta'_{ij}}{n}\right)_{1\times n}. \tag{21}$$

$$\left\{1-\prod_{k=1}^{n}(1-\mathcal{U}_{ki})^{\frac{1}{n}}, \prod_{k=1}^{n}(\mathcal{V}_{ki})^{\frac{1}{n}}\right\} \tag{22}$$

Step 5: Using The mean solution (AV) values, The formula below can be applied to calculate the positive distance from t negative distance from the average (NDA) and the average (PDA).

$$PDA_{ij} = \left(PDA_{ij}\right)_{m\times n} = \frac{max\left(0,\left(\zeta'_{ij}-AV_j\right)\right)}{AV_j} \tag{23}$$

$$NDA_{ij} = \left(NDA_{ij}\right)_{m\times n} = \frac{max\left(0,\left(AV_j-\zeta'_{ij}\right)\right)}{AV_j} \tag{24}$$

Step 6: Calculate SPi and SNi, the weighted total of PDA and NDA, using the following formula:

$$SP_i = \sum_{j=1}^{o} w_j PDA, NP_i = \sum_{j=1}^{o} w_j NDA \tag{25}$$

Step 7: Normalize the solutions of step (5) as follows:

$$NSP_i = \frac{SP_i}{max(SP_i)}, NSN_i = 1-\frac{SN_i}{max(SN_i)} \tag{26}$$

Step 8: Utilizing the NSP_i and NSN_i, determine the assessment score (AS) for every choice.

$$AS_i = \frac{1}{2}(NSP_i + NSN_i) \tag{27}$$

Step 9: Based on the AS calculation findings, we may rank all of the options; the higher value of BS, the superior option chose.

Figure 1 displays a graphical illustration of the suggested approach.

5. *Illustrative examples*

Based on the suggested method for selecting the top industrial robot [29], we build a new model for creating choices within hyperbolic fuzzy soft set data in this study. The difficulty of choosing the ideal industrial robot for specific pick-and-place assignments jobs that require it to stay away from particular obstacles was examined. The issue informs us that there are five distinct robots: The Cybotech V15 Electric Robot, the Hitachi America Processes Robot, the Cincinnati Mila crone T3-726, and the ASEA-IRB 60/2 are represented by τ_1, τ_2, τ_3 and τ_4, respectively. The following four parameters will be used to select the top industrial robot: The variables χ_1 (Capacity of load), χ_2 (Highest possible tip speed), χ_3 (repeatability), and χ_4 are memory capacity (MC), and with load capacity, maximum tip. The benefits of memory capacity, speed, and the manipulator's reach (where greater values are preferred) and the non-beneficial nature of repeatability (where lower values are preferred).

6. *Computational Results*

To illustrate the efficacy and superiority of the explored procedure, we shall offer a practical MCGDM.

Step 1: Collect the expert information $Q = \left(\zeta_{ij}^{r}\right)_{m\times o} = i = 1,2,\dots..n, j = 1,2,\dots\dots o$ about the robots. The details are provided, accordingly, in Tables 2, 3, and 4.

Table 2: The first expert information

	χ_1	χ_2	χ_3	χ_4
τ_1	(0.8,0.5)	(0.5,0.8)	(0.9,0.2)	(0.8,0.4)
τ_2	(0.7,0.6)	(0.9,0.3)	(0.6,0.5)	(0.9,0.6)
τ_3	(0.7,0.4)	(0.4,0.8)	(0.9,0.3)	(0.8,0.5)
τ_4	(0.8,0.7)	(0.9,0.7)	(0.8,0.7)	(0.6,0.9)

Table 3: The second expert information

	χ_1	χ_2	χ_3	χ_4
τ_1	(0.4,0.9)	(0.6,0.8)	(0.7,0.8)	(0.3,0.8)
τ_2	(0.4,0.7)	(0.3,0.9)	(0.9,0.7)	(0.5,0.8)
τ_3	(0.8,0.7)	(0.6,0.5)	(0.8,0.5)	(0.7,0.6)
τ_4	(0.8,0.6)	(0.4,0.8)	(0.6,0.8)	(0.8,0.7)

Table 4: The third expert information

	χ_1	χ_2	χ_3	χ_4
τ_1	(0.6,0.5)	(0.7,0.6)	(0.8,0.5)	(0.7,0.5)
τ_2	(0.9,0.6)	(0.8,0.7)	(0.7,0.9)	(0.6,0.9)
τ_3	(0.4,0.8)	(0.6,0.7)	(0.8, 0.6)	(0.9,0.3)
τ_4	(0.6,0.6)	(0.7,0.9)	(0.8,0.8)	(0.7,0.7)

Step 2: We employ known weights derived from expert opinion data. Weights allocated to experts based on their professional judgements are essential to the process of making decisions. They are utilised in future computations and analyses to establish the decision-making framework.
Step 3: In HYFSS, the information is computed using the Einstein aggregation operator (17). This operator accounts for fuzziness in input by combining their weights in Table 5.

Table 5: weighted matrix

	χ_1	χ_2	χ_3	χ_4
τ_1	(0.6002,0.607616)	(0.636017,0.696663)	(0.80191,0.495596)	(0.632894,.558492)
τ_2	(0.776568,0.629148)	(0.733659,0.656266)	(0.767389,0.752372)	(0.667272,0.806744)
τ_3	(0.613721,0.678919)	(0.564387,0.6547)	(0.825338,0.499423)	(0.838477,0.414622)
τ_4	(0.714286,0.619347)	(0.693771,0.828811)	(0.751752,0.779587)	(0.717204,0.738533)

Step 4: To compute the score value in a hyperbolic fuzzy soft set in table 6. The formula of score value is $\mathbb{S}_{pe} = \frac{1+(\Xi)^2-(\Lambda)^2}{2}$,such that $\mathbb{S}_{pe} \in [0,1]$.

Table 6: Score values

	χ_1	χ_2	χ_3	χ_4
τ_1	(0.495495)	(0.459589)	(0.698722)	(0.54432)
τ_2	(0.603615)	(0.553785)	(0.511411)	(0.397208)
τ_3	(0.457861)	(0.444951)	(0.71588)	(0.765566)
τ_4	(0.563307)	(0.397195)	(0.478688)	(0.484476)

Step 5: 5 We do this by applying the previously computed criterion weights to the HYFFSS hiding layer. The respective weights for each of the criteria for the hidden layer are 0.2, 0.3, 0.35, and 0.15, respectively, to their relative contribution or influence. Each one is a measure of importance of an input. feature or standard in the HYFFSS contributing to decision or overall output.
Step 6: The hidden layer data is combined using the Einstein aggregation operator (17) and weights to generate data from the output layer. When merging hidden layer data, this operator considers the importance of each weight, resulting in a precise and dependable final output.
Step 7: Equation (20) calculates the average solution based on all parameters.

$$AV_1 = (0.684886,0.633189)$$
$$AV_2 = (0.662777,0.705749)$$
$$AV_3 = (0.788581,.617267)$$
$$AV_4 = (0.726695,0.609458)$$

Step 8: To Determine the positive deviation from the mean. $PDA_{ij} = \left(PDA_{ij}\right)_{4\times4}$ matrix and the negative deviation from the mean $NDA_{ij} = \left(NDA_{ij}\right)_{4\times4}$matrix, use Equations (21) and (22), respectively.

$$PDA_{ij} = \left(PDA_{ij}\right)_{4\times4} \begin{pmatrix} 0 & 0 & 0.308296 & 0.019193 \\ 0.130216 & 0.130216 & 0 & 0 \\ 0 & 0 & 0.340423 & 0.433457 \\ 0.054743 & 0 & 0 & 0 \end{pmatrix}.$$

$$NDA_{ij} = \left(NDA_{ij}\right)_{4\times 4}\begin{pmatrix} 0.072228 & 0.13946 & 0 & 0 \\ 0 & 0 & 0.042426 & 0.256262 \\ 0.142695 & 0.166868 & 0.340423 & 0.433457 \\ 0 & 0.256287 & 0.103699 & 0.092861 \end{pmatrix}.$$

Step 9: Calculate the PDA and NDA weighted sum for each option using Equations (23) as illustrated below:

$$Sp_1 = 0.095368$$
$$Sp_2 = 0.038963$$
$$Sp_3 = 0.167145$$
$$Sp_3 = 0.010949$$
$$NS_{p1} = 0.063257$$
$$NS_{p2} = 0.051167$$
$$NS_{p3} = 0.086943$$
$$NS_{p4} = 0.134739$$

Step 10: Normalise the SPi and SNi values for all possibilities using Equations (24) as illustrated below:

Step 11: To calculate the appraisal score AS_i $(i = 1, 2, 3, 4)$ for all alternatives, use Equation (25).

$$AS_1 = 0.5505$$
$$AS_2 = 0.4267$$
$$AS_3 = 0.6774$$
$$AS_4 = 0.0328$$

Step 12: After calculating the aggregated scores for each alternative with the Hyperbolic Fuzzy Soft Set framework, we compute the score values based on the positive and negative distances from the average answer. The choices are subsequently placed in descending order. of aggregated appraisal scores. The option with the greatest score is deemed the most appropriate choice.

$$AS_3 > AS_1 > AS_2 > AS_4$$

7. *Comparative analysis*

Here, we compare the effectiveness of the suggested paradigm for decision-making that is predicated on the hyperbolic fuzzy soft set framework using the Einstein aggregation operator and the EDASS method to several recognized methods for MCDM (multi-criteria decision-making), including COCOSO, GRA, WASPAS, and TOPSIS. The evaluation was conducted using a unified decision matrix with four criteria assigned weights of 0.2, 0.35, 0.30, and 0.15 based on expert advice. The performance of each strategy was compared and scored accordingly. The EDASS method of proposed alternative ranked A3 as the best one successively. demonstrating its ability to accommodate uncertainty and reflect the decision-makers' preferences accurately. Although other methods identified A3 as a top candidate, EDASS exhibited more stability and discrimination power in separating alternatives. These results confirm the robustness and real-world applicability of the proposed model compared to other widely recognized MCDM methods. The comparative outcomes are presented in Table 7.

Table 7: Comparison result

	τ_1	τ_2	τ_3	τ_4
Proposed method	0.5505	0.4267	0.6774	0.0328
COCOSO	1.335605	1.2992	1. 374772	1.207301
GRA	0.51126	0.550514	0.567861	0.451052
WASPAS	0.546927	0.525459	0.568399	0.466026
TOPSIS	0.612828	0.580059	0.755001	0.397311

Table 8: Ranking of comparison result

Methods	Ranking
Proposed method	$\tau_3 > \tau_1 > \tau_2 > \tau_4$
COCOSO	$\tau_3 > \tau_1 > \tau_2 > \tau_4$
GRA	$\tau_3 > \tau_2 > \tau_1 > \tau_4$
WASPAS	$\tau_3 > \tau_1 > \tau_2 > \tau_4$
TOPSIS	$\tau_3 > \tau_1 > \tau_2 > \tau_4$

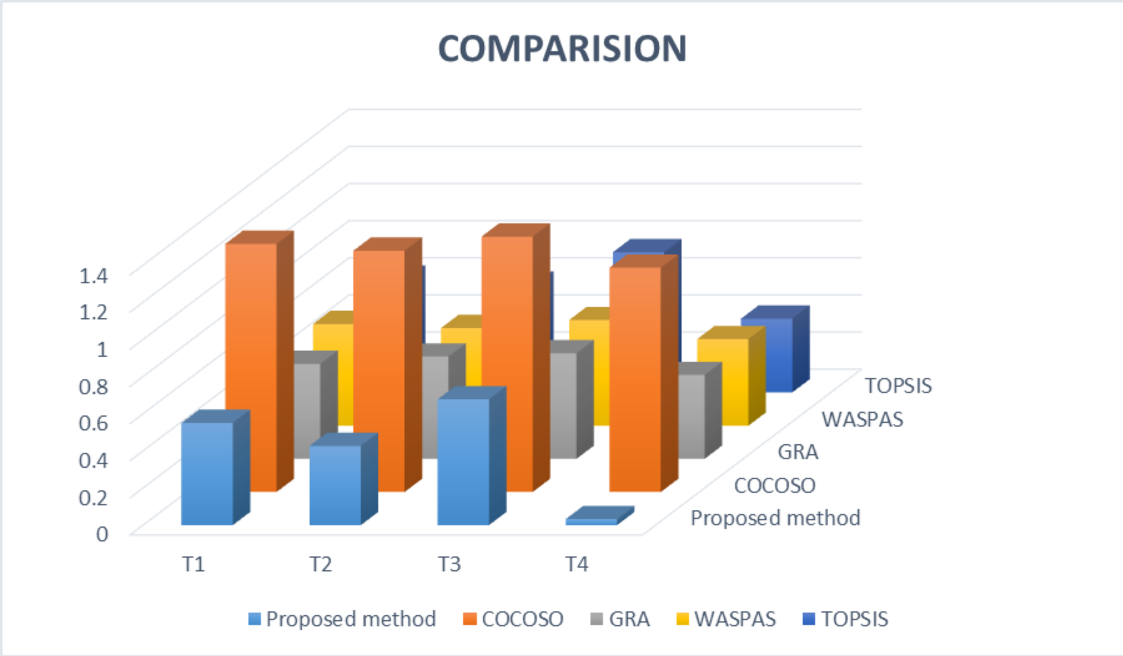

Figure 2: comparative analysis

8. Conclusions

In this paper, we initially considered the concept of hyperbolic fuzzy soft sets and made the theoretical framework for their operations and structure. Subsequently, we extended the system by formulating crucial operational rules and including aggregation operators for hyperbolic fuzzy soft sets. Additionally, we formulated root theorems which form the foundation of this system. In an attempt to establish whether our approach can be applied in real-world, we constructed a robust decision model based on evaluation by distance from average solution, or EDAS for short. This model was employed to solve a real-world problem selection of a robot with information obtained from three experts with individual weights assigned to each. Combining this expert information resulted in a final decision result. Comparative analysis was applied to verify correctness and reliability of our proposed model. The results support the effectiveness of employing hyperbolic fuzzy soft sets in decision-making situations and propose their applicability in more extended scenarios. Comparative analysis confirmed the accuracy and reliability of the proposed model.

In later work, this model will be expanded by continuing its development to include neural networks and the Hamacher aggregation operator for improved decision accuracy construction. Furthermore, the model is generalized to intuitionistic hyperbolic fuzzy soft sets, Pythagorean hyperbolic fuzzy soft sets, and q-rung orthopair hyperbolic fuzzy soft sets. These improved fuzzy structures will improve the representation of uncertainty and hesitation in complex contexts, resulting in a more resilient and adaptable approach to real-world decision-making scenarios in uncertain and multi-criteria systems. The proposed model involves relatively higher computational complexity compared to some classical methods. The performance of the method may depend on the appropriate selection of parameters and aggregation operators. The model does not yet consider dynamic or time-dependent decision environments.

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