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A Novel Fuzzy Intelligence-Based Decision Support System and Its Application

Salman Ahmad[1] and Sadia Zaman[1,*]

[1]Department of Mathematics, Abdul Wali Khan University, Mardan, Mardan ,23200, KP, Pakistan

[*]Corresponding author: sadiazaman317@gmail.com

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ABSTRACT

Fuzzy credibility hyper soft set is a mathematical tool for dealing with uncertain or ambiguous information where the degree of membership of an element to a set is represented by a credibility function. The credibility function assigns a value between 0 and 1 to each element of the universe of discourse, indicating the degree to which the element belongs to the set. The higher the value of the credibility function, the more credible the element is considered to belong to the set. In many real-world applications, partial or unclear data is present, which is why fuzzy credibility hyper soft set theory is able to manage it. Fuzzy credibility hyper soft set could be uses in daily life. It may be used in many different contexts, including decision-making, expert systems, pattern recognition, and image processing. Basic operational rules were first introduced, and then new concepts like fuzzy credibility hyper soft weighted average were developed based on these operational laws. To demonstrate the validity of the described work, an algorithm is set up in the context of F_c^hss and a numerical example is provided.

Keywords

Fuzzy credibility hyper soft set; Aggregation operators; Decision support system.

1. Introduction

A fuzzy set is well-known to be represented by a membership degree that falls between the [0, 1] range [1] . The fields of decision analysis, risk assessment, economics, and prediction are just a few of the areas in which it has been extensively applied. Fuzzy multi-attribute decision-making (MADM) methods have been employed to evaluate choices in relation to the given attributes. Especially in the engineering and management fields, either by one decision maker, a few decision makers, or a few decision makers. Here, the acceptability of alternatives over qualities and the significant weights of attributes can be determined by fuzzily assigned numbers or values [2] − [3]. The problem of fuzzy MADM has been addressed in a variety of ways [4] − [7]outranking methods [8] − [9] distance-based methods [10] − [11] and pairwise comparisons-based methods [11] − [13] Also, other techniques have been reviewed and compared in[14] − [19]. In more recent times, several fuzzy techniques have been introduced and applied to problems with decision-making and evaluation in the fuzzy environment. [20] − [27]. Existing fuzzy MADM methods, however, only indicate fuzzy assessment values and lack the degrees of credibility regarding the fuzzy assessment values in the appropriate assessment of alternatives over attributes because human cognitions and judgements for difficult MADM problems are hazy and uncertain. MADM challenges are frequently rooted in human subjective evaluations and judgements in confusing and unclear circumstances. As a result Experts or decision-makers are more familiar with some attributes than others, they may issue fuzzy evaluation values for those features with varied degrees of trustworthiness [28]. For instance, in certain article review systems, each reviewer or expert is required to rate each paper with a credibility score between 1 and 10, along with their overall evaluation. Assume that an expert or reviewer specifies 6 (corresponding to a fuzzy evaluation value of 0.6) and 0.9 (corresponding to a level or degree of credibility of 0.9) in this situation because the expert's knowledge and/or experience is insufficient, ambiguous, or uncertain. Because of The credibility degree of 0.8 and the fuzzy evaluation value of 0.6 in the pair of fuzzy values are both obvious (0.6, 0.9) are strongly associated, which increases the credibility level of the reviewer's overall evaluation of the text. As Zadeh classical fuzzy notion simply signals without considering its degree of credibility, a fuzzy degree, it is unable to express the information of the pair of fuzzy values (0.6, 0.9). They are also one of the biggest issues in mine planning and operation for the design and assessment of slope designs. So they suggest that you have a thorough understanding of the geology, which is usually convoluted, confusing, and imprecise in terms of the structural and material properties of orebodies. In the meanwhile, designers and decision-makers could lack some expertise and understanding when evaluating slope design schemes based on certain indices or features.

Under uncertain and unpredictable circumstances, human judgements might not always be totally dependable and correct, thus in order to preserve the credibility levels and degrees of the fuzzy evaluation values further provide fuzzy evaluation values. But also show their degrees of credibility. To make the assessment information more plentiful and trustworthy, the fuzzy evaluation the relationship between ideals and credibility levels/degrees in ambiguous and uncertain contexts. Different fuzzy credibility MADM [29] – [32] This paper advances the fuzzy notion by introducing the idea of a fuzzy credibility number (FCN). A pair of fuzzy values expresses both a fuzzy value and a credibility degree. In the next section, we introduce the FCN weighted arithmetic averaging (FCNWAA) and FCN weighted geometric averaging (FCNWGA) operations and FCN scoring function for rating FCNs, and FCN operations. [33] – [35].

There are some draw backs of fuzzy credibility number

Lack of Accuracy: Fuzzy credibility scores are innately approximate and cannot give accurate values. As a result, they could be less helpful when exact values are required.

Subjectivity: Fuzzy credibility metrics rely on subjective evaluations of a variable's degree of membership in a certain fuzzy collection. This subjectivity may result in contradictions and mistakes.

Restricted Applicability: Situations with a lot of uncertainty or ambiguity benefit the most from fuzzy credibility numbers. More accurate numerical techniques, however, could be more suited in many circumstances.

Complexity: For people who are unfamiliar with the mathematical concepts that underlie fuzzy logic and fuzzy credibility numbers, they can be complicated and challenging to grasp. Non-experts may find it challenging to use and comprehend the data as a result.

Lack of Universality: Fuzzy credibility scores may not be suited for all situations or decision-making processes and are not always relevant. Moreover, they might not be compatible with other decision-making frameworks or tools.

Contribution of Work: To overcome the above draw backs we introduced fuzzy credibility hyper soft set. A sort of mathematical set known as a hyper soft set enables more flexible representation of uncertain or fuzzy information. Each component of this type of set receives a degree of membership, which denotes how likely or credible it is that the element is a part of the set. The set itself is also given a level of credibility, which expresses the degree of assurance or trust that the set properly represents the underlying data. The term "hyper soft" alludes to the fact that this kind of set provides for even more flexibility and granularity than conventional fuzzy sets, since it allows the usage of numerous degrees of membership and credibility for each element. Uncertainty regarding credibility As they offer a more nuanced and flexible method of describing complicated or ambiguous information, hyper soft sets have applications in a range of domains, including decision-making, pattern identification, and data analysis. We use some basic concept and aggregation operators. Fuzzy credibility hyper soft set, average aggregation operators, basic notation of F_c^hSWA, F_c^hSOWA and F_c^hSHA. A detailed discussion of the attributes of operators follows the elaboration of the operators.

Paper Arrangement: The rest of the paper is organized is follows; in portion 2 we define some basic definition of fuzzy credibility hyper soft set. In portion 3 we discuss operational laws for fuzzy Credibility hyper soft set. In portion 4 we discuss aggregation operators based on fuzzy credibility hyper soft set. In this portion 5 we will research a novel MCDM approach based on (F_c^hWA), (F_c^hOA) and(F_c^hHA) using aggregation operators to address MCDM issues in a FCHSS Also talk about how to use the suggested method. In portion 6 we present ranking of the outcome based on our proposed method.

2. *Preliminaries*

This section provides the basic concept and fundamental result for the remaining paper.

Definition 1: A non-empty set called a fuzzy set (FS) is one where U is provided by

$$F = \{x, b(x): x \in \text{U} \}, \tag{1}$$

where $b: U \to [0,1]$ denote the membership grade MG.

Definition 2: Let E be a collection of parameters such that FE, and let U be a universal set. A pair (H, T), where H is the map provided by H, is called to be soft set (SS) over U. The power set of U is T:→P(U).

Definition 3. Let H be a set of parameters and U be a universal set. A pair (T, H) is called fuzzy soft set over U. Where T is the map given by T:H →Fs^u

$$\text{P}\ (x_i) =\{x_{i,}|a_j(x_i) : x_i \in U\}, \tag{2}$$

where Fs^u is the family of all FS_s on U where$a_j(x_i)$ represent the member degree MD Condition are satisfying that $0 \leq\ a\ (x)\ +\ b\ (x) \leq 1$.

Definition 4: Let U be the Universal set, followed by a fuzzy credibility hyper soft set (F_c^hS) over U is given by

$$\text{S}\wp_{ij}(xi) = \{\ (xi), \mathcal{L}_j(x_\text{i})(C_j(x_\text{i})) : x_\text{i} \in \text{U}\}, \tag{3}$$

where F_c^hS is the family of F_c^hNs over $\mathbb{U}$. Here $\left(\mathcal{L}_j(x_\text{i})\right)$ and $(C_j(x_\text{i}))$ represents the MG and degree of credibility respectively condition are satisfying $0 \leq \mathcal{L}(x) + c(x) \leq 1$.

Definition 5: Let U be a global set and E be a collection of parameters. And $T\ \subseteq\ E$. A pair (S, T) is called to be fuzzy credibility hyper soft set F_c^hS over $\mathbb{U}$, where "S" is the map given by S: T → F_c^hS which is define as,

$$\text{S}\wp_{ij}(xi) = \left\{x_\text{i}, \left(\mathcal{L}_j(x_\text{i})\right), (C_j(x_\text{i})) : x_\text{i} \in \text{U}\right\}, \tag{4}$$

where F_c^hS is the family of F_c^hS over $\mathbb{U}$. Here $\left(\mathcal{L}_j(x_\text{i})\right)and(C_j(x_\text{i}))$ represents the MG and degree of credibility respectively satisfying as condition $0 \leq \mathcal{L}\big(x(x)\big) + c(x) \leq 1$.

For simplicity $\left\{x_\text{i}, \left(\mathcal{L}_j(x_\text{i})\right), (C_j(x_\text{i}))\right\}$ is called fuzzy credibility hyper soft number (F_c^hNs). Also refusal grade is defined as $\text{S}\wp_{ij} = 1\text{-}\left(\mathcal{L}_j(x_\text{i}) + C_j(x_\text{i})\right)$.

Definition 6: Suppose $S_{1_{\wp ij}}$= $(\mathcal{L}_{ij}$,$C_{ij})$ and $S_2\wp_{ij}$=$(\mathcal{L}_{ij}$,$C_{ij})$ be two F_c^hNs and $\lambda > 0$. Then, fundamental operational rules for F_c^hNs are defined by

1. $S_{1\wp ij}$⊕$S_{2\wp ij}$=$\left(\mathcal{L}_{1ij}\right) + \left(\mathcal{L}_{2ij}\right) - \left(\mathcal{L}_{1ij}\right)\left(\mathcal{L}_{2ij}\right)$, $\left(C_{1ij}\right)\left(C_{2ij}\right)$
2. $S_{1\wp ij}$⊗$S_{2\wp ij}$= $(\mathcal{L}_{1ij}\mathcal{L}_{2ij}$,$C_{1ij}C_{2ij})$
3. $\lambda S_{\wp ij}$= $(1-\left(1 - \mathcal{L}_{ij}\right)^{\lambda})$, ${C_{ij}}^{\lambda}$
4. $S_{\wp ij}^{\lambda}$= $(\mathcal{L}_{ij}^{\lambda}$,$C_{ij}^{\lambda})$

Definition 7: S_1= $(\mathcal{L}_1, C_1)$ and $S_{2=}(\mathcal{L}_{2,}C_2)$ be two F_c^hNs and λ >0. Then basic operation on F_c^hNs are define by

1. S_1⊗S_2= $((\mathcal{L}_1(x)\mathcal{L}_2(x)), (\text{C}_1(x) + C_2(x)) - (\text{C}_1(x)\text{C}_2(x))$
2. S_1⊕S_2= $((\mathcal{L}_1(x) + \mathcal{L}_2(x)$ $-(\mathcal{L}_1(x)(\mathcal{L}_2(x)), (\text{C}_1(x)C_2(x)$
3. S_1^{λ} = $((\mathcal{L}_1(x)^{\lambda}$, $(1-(1 - C_1(x)^{\lambda}$.
4. λS_1= $((1-(1 - \mathcal{L}_1(x)^{\lambda})$, $C_1(x)^{\lambda})$

Example 1: Suppose $S\mathcal{p}_{11} = (0.2, 0.4)$, $S\mathcal{p}_{12}$=(0.3,0.6) and S=(0.2,0.5) be two F_c^HNs and $\lambda = 2$.

1. $S\mathcal{p}_{11} \cup S\mathcal{p}_{12}$=$\langle \max(0.2,0.3), \min(0.4,0.6)\rangle$ =(0.3,0.4)
2. $S\mathcal{p}_{11} \cap S\mathcal{p}_{12}$=$\langle \min(0.2,0.3), \min(0.4,0.6)\rangle$ =(0.2,0.4)
3. S^c=(0.5,0.2)
4. $S\mathcal{p}_{11} \oplus S\mathcal{p}_{12}$=$(0.2 + 0.3 - (0.2)(0.3)), ((0.4)(0.6))$ = (0.54,0.24)
5. $S\mathcal{p}_{11} \otimes S\mathcal{p}_{12}$=((0.2)(0.3),(0.4)(0.6)) = (0.06,0.24)
6. λS=(1-$(1-0.2)^2$,$(0.5)^2$) = (0.36,0.25)
7. S^λ = $((0.2)^2, (0.5)^2)$ = (0.04,0.25)

Definition 8: For a F_c^hNs, $S\mathcal{p}_{ij} = (\mathcal{L}_{ij}, C_{ij})$, the definitions of the score function (SF) and accuracy function (AF), as follows,

$$Sc(S\mathcal{p}_{ij}) = \frac{(1+\mathcal{L}_{ij}-C_{ij})}{2}, \text{SF } (S\mathcal{p}_{11}) \in [-1, 1] \quad (5)$$

$$Ac(S\mathcal{p}_{11}) = \mathcal{L}_{ij} - C_{ij},, \text{Ac } (S\mathcal{p}_{11}) \in [-1, 1] \quad (6)$$

Definition 9: For the collection of F_c^hSNs $S\mathcal{p}_{ij} = (\mathcal{L}_{ij}, C_{ij})$, where $i = 1,2,3, \dots, n$ and $j = 1,2,3, \dots, m$, if $w = \{w_1, w_2, w_3, \dots, w_n\}$. Represents the weight vector (WV) of e_i experts and $\mathcal{p}$= $\{\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \dots, \mathcal{p}_m\}$ represent the (WV) of parameters $\mathcal{p}_j$ and condition $w_i\, \mathcal{p}_j \in [0, 1]$ with $\sum_i^n w_i = 1$ and with $\sum_j^m \mathcal{p}_j = 1$, then F_c^hSWA operators is the mapping define as F_c^hSWA:$\mathcal{R}^n \rightarrow \mathcal{R}$, where ($\mathcal{R}$ is the family of all F_c^hSNs).

$$F_c^h\text{SWA}(S\mathcal{p}_{11}, S\mathcal{p}_{12}, S\mathcal{p}_{13}, \dots, S\mathcal{p}nm) = \oplus_{j=1}^m \mathcal{p}_j(\oplus_i^n w_i S\mathcal{p}_{ij})$$

Theorem 1: For a collection of F_c^hSNs where $S\mathcal{p}_{ij}$= $(\mathcal{L}_{ij}, C_{ij})$, then a F_c^hSNs is again the aggregated result for the F_c^hSWA, operator, as shown by,

$$F_c^h\text{SWA}(S\mathcal{p}_{11}, S\mathcal{p}_{12}, S\mathcal{p}_{13}, \dots, S\mathcal{p}nm) = \oplus_{j=1}^m \mathcal{p}_j(\oplus_i^n w_i S\mathcal{p}_{ij})$$

$$= 1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - (\mathcal{L}_{ij})^{w_i}\right) \mathcal{p}_j\right), \prod_{j=1}^m \left(\prod_{i=1}^n (C_{ij})^{w_i}\ \mathcal{p}_j\right), \quad (7)$$

where $w = \{w_1, w_2, w_3, \dots, w_n\}$.represent the weight vector (WV) of e_i experts and $\mathcal{p}$= $\{\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \dots, \mathcal{p}_m\}$, represent the (WV) of parameters $\mathcal{p}_j$ as condition, $w_i\, \mathcal{p}_j \in$[0,1] with $\sum_i^n w_i$=1 and with $\sum_j^m \mathcal{p}_j$=1.

Proof: This finding should be applied using the mathematical induction approach.

Using operational laws, it can be seen that

$S_1 \oplus S_2$= $((\mathcal{L}_1(x) + \mathcal{L}_2(x) - (\mathcal{L}_1(x)(\mathcal{L}_2(x)), (C_1(x) C_2(x))$

λS_1= $((1-(1-\mathcal{L}_1(x)^\lambda), (C_1(x)^\lambda)$ for $\lambda \geq 1$.

We'll illustrate Eq (7) is true for n=2 and m=2.

$$F_c^h\text{SWA } (S\mathcal{p}_{11}, S\mathcal{p}_{12}, S\mathcal{p}_{13}, \dots S\mathcal{p}nm) = \oplus_{j=1}^2 \mathcal{p}_j(\oplus_{i=1}^2 S\mathcal{p}_{ij})\, w_i$$

$$= \mathcal{p}_1(\oplus_{i=1}^2 w_i S\mathcal{p}_{ij}) \oplus \mathcal{p}_2(\oplus_{i=1}^2 w_i S\mathcal{p}_{ij})$$

$$= (1 - \prod_{j=1}^2 \left(\prod_{j=1}^2 \left(1 - (\mathcal{L}_{ij})^{w_i}\right) \mathcal{p}_j\right), \prod_{j=1}^2 \left(\prod_{j=1}^2 \left((C_{ij})^{w_i}\right) \mathcal{p}_j\right)$$

The conclusion is valid for $n = 2$ then $m = 2$.

Next, think about Eq (7) is true n =$\mathbb{K}_1$ then $m = \mathbb{K}_2$

$$F_c^h\text{SWA } (S\mathcal{p}_{11}, S\mathcal{p}_{12}, S\mathcal{p}_{13}, \dots S\, \mathcal{p}_{j_{\mathbb{K}_1\mathbb{K}_2}}) = \oplus_{j=1}^{\mathbb{K}_2} \mathcal{p}_j(\oplus_i^{\mathbb{K}_1} w_i S\mathcal{p}_{ij})$$

$$= 1 - \prod_{j=1}^{\mathbb{K}_2} \left(\prod_{i=1}^{\mathbb{K}_1} \left(1 - (\mathcal{L}_{ij})^{w_i}\right) \mathcal{p}_j\right), \prod_{j=1}^{\mathbb{K}_2} \left(\prod_{i=1}^{\mathbb{K}_1} \left((C_{ij})^{w_i}\right) \mathcal{p}_j\right)$$

Furthermore, suppose that Eq(7) is true for $n = \mathbb{K}_{1+1}$ and $m = \mathbb{K}_{2+1}$

$$F_c^h\text{SWA } (S\mathcal{p}_{11}, S\mathcal{p}_{12}, S\mathcal{p}_{13}, \dots, S\, \mathcal{p}_{j_{(\mathbb{K}_{1+1}\ \mathbb{K}_{1+2})}}) = \left\{(\oplus_{j=1}^{\mathbb{K}_2} \mathcal{p}_j(\oplus_i^{\mathbb{K}_1} w_i S\mathcal{p}_{ij})\right\} \oplus \mathcal{p}_{(\mathbb{K}_{1+1})} \left(w_{(\mathbb{K}_{2+1})} S\mathcal{p}_{\mathbb{K}_{1+1}\ \mathbb{K}_{2+1}}\right)$$

$$= \left(1-\prod_{j=1}^{\mathbb{K}_2}\left(\prod_{j=1}^{\mathbb{K}_1} 1\left(1-\left(\mathcal{L}_{ij}\right)^{w_i}\right) p_j\right), \prod_{j=1}^{\mathbb{K}_2}\left(\prod_{j=1}^{\mathbb{K}_1} 1\left(\left(C_{ij}\right)^{w_i}\right) p_j\right)\right)$$

$$\oplus p_{(\mathbb{K}_{1+1})}\left(w_{(\mathbb{K}_{2+1})} \mathrm{S}p_{\mathbb{K}_{1+1}\ \mathbb{K}_{2+1}}\right)$$

$$= \left(1-\prod_{j=1}^{\mathbb{K}_{2+1}}\left(\prod_{j=1}^{\mathbb{K}_{1+1}} 1\left(1-\left(\mathcal{L}_{ij}\right)^{w_i}\right) p_j\right), \prod_{j=1}^{\mathbb{K}_{2+1}}\left(\prod_{j=1}^{\mathbb{K}_{1+1}} 1\left(\left(C_{ij}\right)^{w_i}\right) p_j\right)\right).$$

It is obvious from the equation above that the aggregate value is also F_c^hSNs. As a result, provided Eq(1) is accurate for $n=\mathbb{K}_1+1$ and $m=\mathbb{K}_2+1$.As a result it is clear for all $m\ n\ \geq 1$.

Theorem 2: Let $\mathrm{S}p_{ij}=(\mathcal{L}_{ij}, C_{ij})$ be the family of $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$, F_c^hNs, $w=\{w_1,w_2,w_3,\ldots,w_n\}$represent the WV of ei experts and $p=\{p_1,p_2,p_3,\ldots,p_m\}^t$ represent the WV of parameters ρj with condition $w_i, p\, j \in [0,1]$ and $\sum_{j=1}^{m} \mathrm{p}_j = 1$ with $\sum_{i=1}^{n} w_i = 1$ and $F_c^h SWA$ t the following properties are holds.

1. **(Impotency):** Let $\mathrm{S}p_{ij}=\left(\mathcal{L}_{ij}, C_{ij}\right)=\mathrm{Sp}$ for all $j=1,2,\ldots,m$, and $i=1,2,\ldots,n$. Where $\mathrm{Sp}=(\mathcal{L},C)$, F_c^hSWA $(\mathrm{S}p_{11},\mathrm{S}p_{12},\mathrm{S}p_{13},\ldots\mathrm{S}p_{nm})=\mathrm{Sp}$

Proof: If $\mathrm{S}p_{ij}=\left(\mathcal{L}_{ij}, C_{ij}\right)=\mathrm{Sp}$ for all $j=1,2\ldots,m$, and $i=1,2,\ldots,n$ where $\mathrm{Sp}=(\mathcal{L},C)$ then from theorem 1, we have, $F_c^h SWA(\mathrm{S}p_{11},\mathrm{S}p_{12},\mathrm{S}p_{13},\ldots,\mathrm{S}p_{nm})$

$$= 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n} 1\left(1-(\mathcal{L}_{ij})^{w_i}\right) p_j\right), \prod_{j=1}^{m}\left(\prod_{i=1}^{n} 1(C_{ij})^{w_i}\ p_j\right)$$

$$= \left(1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n} 1\left(1-(\mathcal{L})^{w_i}\right) p_j\right), \prod_{j=1}^{m}\left(\prod_{i=1}^{n} 1(C)^{w_i}\ p_j\right)\right)$$

$$= (1-(1-(\mathcal{L}),(C)) = (\mathcal{L},C) = Sp$$

Hence $F_c^h SWA\ (Sp_{11}, Sp_{12}, Sp_{13},\ldots,Sp_{nm})=Sp$

2. **(Boundedness):** If $\mathrm{S}p_{ij}{}^{-}(min_j min_i,\{\mathcal{L}_{ij}\}, max_j max_i\{C_{ij}\}$, and $\mathrm{S}p_{ij}{}^{+}=(max_j max_i\{\mathcal{L}_{ij}\}, (min_j min_i\{C_{ij}\})$, then $\mathrm{S}p_{ij}{}^{-}\leq F_c^hSWA(\mathrm{S}p_{11},\mathrm{S}p_{12},\mathrm{S}p_{13},\ldots\mathrm{S}p_{nm})=\mathrm{S}p_{ij}{}^{+}$.

Proof: As $\mathrm{S}p_{ij}{}^{-}(min_j min_i,\{\mathcal{L}_{ij}\}$,$max_j max_i\{C_{ij}\}$, and $\mathrm{S}p_{ij}{}^{+}=(max_j max_i\{\mathcal{L}_{ij}\}, (min_j min_i\{C_{ij}\})$,then we have to prove $\mathrm{S}p_{ij}{}^{-}\leq F_c^hSWA(\mathrm{S}p_{11},\mathrm{S}p_{12},\mathrm{S}p_{13},\ldots\mathrm{S}p_{nm})=\mathrm{S}p_{ij}{}^{+}$.

Now for each $j=1,2,\ldots,m$, and $i=1,2,\ldots,n$

$min_j min_i,\{\mathcal{L}_{ij}\}\leq\{\mathcal{L}_{ij}\}\leq max_j max_i\{\mathcal{L}_{ij}\} \Leftrightarrow 1\text{-}max_j max_i\{\mathcal{L}_{ij}\}\leq 1\text{-}\mathcal{L}_{ij}\leq min_j min_i\{\mathcal{L}_{ij}\}$

$$\Leftrightarrow \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(1-max_j max_i(\mathcal{L}_{ij})^{w_i}\right) p_j\right) \leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1-(\mathcal{L}_{ij})^{w_i}\right) p_j)$$

$$\leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(1-min_j min_i(\mathcal{L}_{ij})^{w_i}\right) p_j\right) \Leftrightarrow \left((1-max_j max_i(\mathcal{L}_{ij})^{\sum_{i=1}^{n} w_i}\right)\sum_{j=1}^{m} p_j\right)$$

$$\leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(1-(\mathcal{L}_{ij})^{w_i}\right) p_j\right) \leq \left((1-min_j min_i(\mathcal{L}_{ij})^{\sum_{i=1}^{n} w_i}\right)\sum_{j=1}^{m} p_j\right)$$

$$\Leftrightarrow \left((1-max_j max_i(\mathcal{L}_{ij})\right) \leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(1-(\mathcal{L}_{ij})^{w_i}\right) p_j\right) \leq \left((1-min_j min_i(\mathcal{L}_{ij})\right)$$

$$\Leftrightarrow 1-\left((1-min_j min_i(\mathcal{L}_{ij})\right) \leq 1-\prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(1-\left(\mathcal{L}_{ij}\right)^{w_i}\right) p_j\right) \leq 1-\left((1-max_j max_i(\mathcal{L}_{ij})\right)$$

Hence $\left((min_j min_i(\mathcal{L}_{ij})\right) \leq 1-\prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(1-\left(\mathcal{L}_{ij}\right)^{w_i}\right) p_j\right) \leq \left(max_j max_i(\mathcal{L}_{ij})\right)$.

Now for each j=1,2,…,m, and i=1,2,…,n we have

$$\left((min_j min_i(\mathcal{C}_{ij})\right) \leq \left(max_j max_i(\mathcal{C}_{ij})\right) \Leftrightarrow \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(1-(\mathcal{C}_{ij})^{w_i}\right) p_j) \leq \prod_{j=1}^{m}\prod_{j=}^{m} 1\left((\mathcal{C}_{ij})^{w_i}\right) p_j)$$

$$\leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(max_j max_i\left(\mathcal{C}_{ij}\right)^{w_i}\right) p_j) \Leftrightarrow \left(min_j min_i\left(\mathcal{L}_{ij}\right)^{\sum_{i=1}^{n} w_i}\right)\sum_{j=1}^{m} p_j\Bigg)$$

$$\leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(\mathcal{C}_{ij})^{w_i}\right) p_j) \leq \left(max_j max_i(\mathcal{C}_{ij})^{\sum_{i=1}^{n} w_i}\right)\sum_{j=1}^{m} p_j\Bigg)$$

$$\Longrightarrow \left(min_j min_i(\mathcal{C}_{ij)}\right) \leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(\mathcal{C}_{ij})^{w_i}\right) p_j\right) \leq max_j max_i\left(\mathcal{C}_{ij}\right).$$

$$\left((min_j min_i(\mathcal{L}_{ij})\right) \leq 1-\prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(1-\left(\mathcal{L}_{ij}\right)^{w_i}\right) p_j\right) \leq \left(max_j max_i(\mathcal{L}_{ij})\right).$$

and $\left(min_j min_i(\mathcal{C}_{ij)}\right) \leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(\mathcal{C}_{ij})^{w_i}\right) p_j\right) \leq max_j max_i\left(\mathcal{C}_{ij}\right).$

$$(\,Sp_{ij}^{-}) \leq F_c^h\text{SWA}(Sp_{11,} Sp_{12,} Sp_{13,} \ldots.. Spnm_{.}) \leq Sp_{ij}^{+}.$$

3. $(\boldsymbol{Monotonicity})$**:** Let $Sp_{ij}{}^{\circ}=(\mathcal{L}_{ij}{}^{\circ}, \mathcal{C}_{ij}{}^{\circ})$ be a different collection of $F_c^h S$Ns for all each i=1,2,…,n and j=1,2,…,m, Implies that $\left(\mathcal{L}_{ij)}\right) \leq \left(\mathcal{L}_{ij}{}^{\circ},\right), \left(\mathcal{C}_{ij)}\right) \geq \left(\mathcal{C}_{ij}{}^{\circ},\right)$, then,

$$F_c^h\text{SWA}\ (Sp_{11,} Sp_{12,} Sp_{13,} \ldots.. Spnm_{.}) \leq F_c^h\text{SWA}\left(Sp_{11,}{}^{\circ} Sp_{12,}{}^{\circ} Sp_{13}{}^{\circ} \ldots \ldots Sp_{nm}{}^{\circ}\right).$$

Proof: $\left(\mathcal{L}_{ij)}\right) \leq \left(\mathcal{L}_{ij}{}^{\circ},\right), \left(\mathcal{C}_{ij)}\right) \geq \left(\mathcal{C}_{ij}{}^{\circ},\right)$, for all each $j = 1,2,\ldots,m$, and $i = 1,2,\ldots,n$ so

$\left(\mathcal{L}_{ij)}\right) \leq \left(\mathcal{L}_{ij}{}^{\circ},\right) \Rightarrow 1-\left(\mathcal{L}_{ij}{}^{\circ},\right) \leq 1-\left(\mathcal{L}_{ij)}\right)$

$$\Rightarrow \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(1-(\mathcal{L}_{ij}{}^{\circ})^{w_i}\right) p_j) \leq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(1-(\mathcal{L}_{ij}{}^{\circ})^{w_i}\right) p_j)$$

$$\Rightarrow 1-\prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(1-\left(\mathcal{L}_{ij}{}^{\circ}\right)^{w_i}\right) p_j\right) \leq 1-\prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\,(1-\left(\mathcal{L}_{ij}{}^{\circ}\right)^{w_i}\right) p_j\right) \quad \mathcal{L}_{\mathfrak{J}_{s\circ}}$$

$$\text{and } \left(\mathcal{C}_{ij)}\right) \geq \left(\mathcal{C}_{ij}{}^{\circ},\right) \Rightarrow \prod_{i=}^{n} 1\left(\mathcal{C}_{ij)}\right)^{w_i} \geq \prod_{i=}^{n} 1\left(\mathcal{C}_{ij}{}^{\circ}\right)^{w_i}$$

$$\Rightarrow \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(\mathcal{C}_{ij)}\right)^{w_i}\right) p_j\right) \geq \prod_{j=}^{m} 1\left(\prod_{i=}^{n} 1\left(\mathcal{C}_{ij}{}^{\circ}\right)^{w_i}\right) p_j\right)$$

Let $\mathfrak{J}_{s\circ}= F_c^h\text{SWA}\left(s\circ_{p_{11,}}\, s\circ_{p_{12,}}\, s\circ_{p_{13,}} \ldots\ldots s\circ_{pnm_{,}}\right)=\left(\mathcal{L}_{\mathfrak{J}_{s\circ}}, \mathcal{C}_{\mathfrak{J}_{s\circ}}\right)$, then from Eq(4) and (5) we obtain, $\mathcal{L}\mathfrak{J}_s \leq \mathcal{L}_{\mathfrak{J}_{s\circ}}$, $\mathcal{C}\mathfrak{J}_s \leq \mathcal{C}_{\mathfrak{J}_{s\circ}}$. $F_c^h\text{SWA}\ (Sp_{11,} Sp_{12,} Sp_{13,} \ldots.. Spnm_{.}) < F_c^h\text{SWA}\left(s\circ_{p_{11,}}\, s\circ_{p_{12,}}\, s\circ_{p_{13,}} \ldots\ldots s\circ_{pnm_{,}}\right)$

3. Methodology

In this part, we'll look at a novel MCDM approach for solving MCDM issues in the context of F_c^hSNs that relies on the aggregation operators (F_c^hSWA), (F_c^hSOWA) and(F_c^hSHWA).We suppose A= $\{x_1, x_2, x_3 \ldots.x_v\}$ be all the set of (v) alternative, E= $\{E_1, E_2, E_3, \ldots.E_n,\}$ be the family of n senior expert with $p = \{p_1, p_2, p_3, \ldots.p_m,\}$ as a family of m parameters. The expert evaluates each $x_1(\ell = 1,2,3,\ldots v)$ according to their respective parameters $p_j(j = 1,2,3\ldots.m)$. Let's say that experts' evaluation information takes the form FCHSNS $Sp_{ij}=\left(\mathcal{L}_{ij}, \mathcal{C}_{ij}\right)$ for i=1,2,…,n and j=1,2,…,m. Let $w = \{w_1, w_2, w_3, \ldots, w_n\}$ and p= $\{p_1, p_2, p_3, \ldots, p_{nm}\}$ are the WVs of "ei" experts and WV of parameters ρj in accordance with the condition $wi, p\, j \in [0,1]$ and with $\sum_{i=1}^{n} w_i$ =1 and $\sum_{j=1}^{m} p_j = 1$. The matrix provides the total information. M=$\left[Sp_{ij}\right]n \times m$.

Step-1: Organize all evaluation data provided by experts for each choice according to their related parameters to create an overall decision matrix, M=$\left[Sp_{ij}\right]n \times m$

$$M=\begin{bmatrix} (\mathcal{L}_{11}, \mathcal{C}_{11})(\mathcal{L}_{12}, \mathcal{C}_{12}) \ldots.(\mathcal{L}_{1m}, \mathcal{C}_{1m}) \\ (\mathcal{L}_{21}, \mathcal{C}_{21})(\mathcal{L}_{22}, \mathcal{C}_{22}) \ldots.(\mathcal{L}_{2m}, \mathcal{C}_{2m}) \\ \ldots\ldots.. \\ (\mathcal{L}_{n1}, \mathcal{C}_{n1})(\mathcal{L}_{n2}, \mathcal{C}_{n2}) \ldots.(\mathcal{L}_{nm}, \mathcal{C}_{nm}) \end{bmatrix}$$

Step- 2: The F_c^hSNs decision matrix that is provided in Step-1 should be normalized because it contains two types of parameters, cost type parameters and benefit type parameters, if necessary to the following formula is used.

$$\wp_{ij} = \begin{Bmatrix} S^c \wp_{ij} = \text{for cost type parameters} \\ S\wp_{ij} = \text{for benefit type parameter} \end{Bmatrix},$$

where $S^c \wp_{ij} = (\mathcal{L}_{ij}, C_{ij})$ denote the complement of $S\wp_{ij} = (\mathcal{L}_{ij}, C_{ij})$.

Step- 3: Aggregate the F_c^hSNs with the recommended aggregate AoPs for each parameter $\wp_1$ $(\ell = 1,2,3, \dots, v)$ to obtain $P_1 = (\mathcal{L}_1, C_1)$.

Step- 4: Calculating the score values for each P_1.

Step-5: Ranking the outcomes for each alternative $x_1(\ell = 1,2,3, \dots, v)$ and select the best outcome. Figure 1 show flowchart of proposed model.

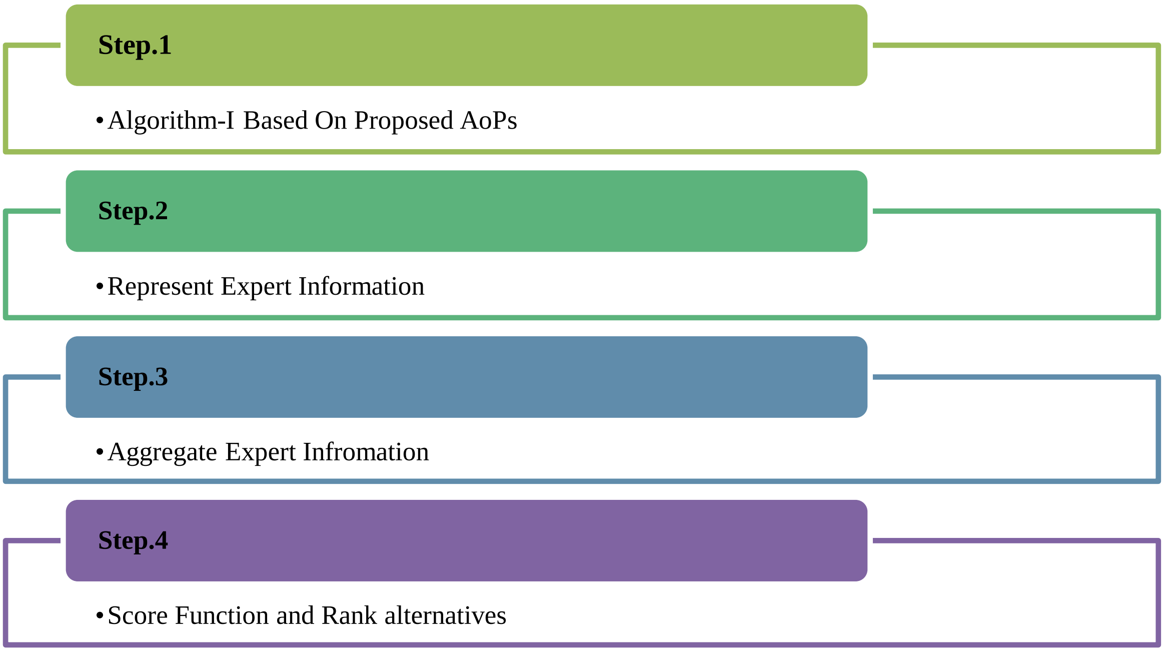

Figure 1: flowchart of proposed model

4. *Illustrative examples*

To show how valuable the current work is, we provide a full explanation of the above-mentioned method in this part along with a representative example. Consider a situation in which a person is trying to decide between four different type brands. B= $\{x_1 = Dunlop, x_2 = MRF\ tyres, x_3 = Bridgestone, x_4 = Hankook\}$. Let a group of five experts decide. $E = \{E_1, E_2, E_3, E_4, E_5\}$ with WVs w=(0.14, 0.11,0.28,0.22,0.25) and having parameters P, follows as $p_1 =$ Fuel consumption, p_2 =Aquaplaning, $p_3 =$ Cornering grip, $p_4 =$ Internal noise, $p_5 =$ Durability. With WVs p={0.17,0.25,0.10,0.29,0.19} in the shape of F_c^hSNs. Now, we choose the best alternative using the suggested algorithm.

5. *Computational Results*

Step- 1: Table-1, Table-2, Table-3 and Table-4 show all of the expert knowledge based on the F_c^hSNs.

Step- 2: A normalizer is not required for the F_c^hSNs matrix when the parameters are identical.

Step- 3: Applying Eq (7) to each option $x_i(i = 1,2,3,4)$.

Step-4: The score values and ranking results for each alternative is as follows.

Table-5: Ranking of all alternatives by different proposed methods.

Table: F_c^hSNs Decision Matrix for alternative x_1

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$
$\mathbf{E_1}$	(0.2,0.2)	(0.1,0.3)	(0.2,0.3)	(0.4,0.2)	$(\mathbf{0.8}, \mathbf{0.1})$
$\mathbf{E_2}$	(0.3,0.2)	(0.3,0.1)	(0.5,0.1)	(0.7,0.2)	$(\mathbf{0.5}, \mathbf{0.1})$
$\mathbf{E_3}$	(0.4,0.1)	(0.5,0.4)	(0.5,0.4)	(0.3,0.2)	$(\mathbf{0.5}, \mathbf{0.4})$
$\mathbf{E_4}$	(0.5,0.3)	(0.5,0.2)	(0.6,0.3)	(0.1,0.2)	$(\mathbf{0.6}, \mathbf{0.3})$
$\mathbf{E_5}$	(0.3,0.2)	(0.2,0.1)	(0.4,0.3)	(0.6,0.4)	$(\mathbf{0.6}, \mathbf{0.2})$

Table 2: $F_c^h SNs$ Decision Matrix for alternative x_2

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$
$\mathbf{E_1}$	(0.6,0.4)	(0.1,0.2)	(0.4,0.4)	(0.2,0.3)	(0.6,0.1)
$\mathbf{E_2}$	(0.5,0.3)	(0.6,0.3)	(0.6,0.2)	(0.5,0.1)	(0.1,0.2)
$\mathbf{E_3}$	(0.5,0.1)	(0.5,0.2)	(0.4,0.1)	(0.1,0.3)	(0.2,0.1)
$\mathbf{E_4}$	(0.2,0.3)	(0.7,0.1)	(0.4,0.3)	(0.2,0.3)	(0.2,0.2)
$\mathbf{E_5}$	(0.7,0.3)	(0.2,0.3)	(0.3,0.8)	(0.1,0.2)	(0.3,0.2)

Table 3: $F_c^h SNs$ Decision Matrix for alternative x_3

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$
$\mathbf{E_1}$	(0.1,0.2)	(0.4,0.1)	(0.2,0.3)	(0.1,0.3)	(0.3,0.3)
$\mathbf{E_2}$	(0.3,0.2)	(0.7,0.3)	(0.8,0.1)	(0.2,0.7)	(0.2,0.5)
$\mathbf{E_3}$	(0.4,0.2)	(0.3,0.6)	(0.4,0.5)	(0.3,0.6)	(0.8,0.3)
$\mathbf{E_4}$	(0.3,0.3)	(0.4,0.3)	(0.2,0.8)	(0.2,0.5)	(0.5,0.7)
$\mathbf{E_5}$	(0.1,0.3)	(0.5,0.4)	(0.8,0.4)	(0.6,0.7)	(0.4,0.3)

Table 4: $F_c^h SNs$ Decision Matrix for alternative x_4

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$
$\mathbf{E_1}$	(0.1,0.2)	(0.2,0.1)	(0.5,0.3)	(0.4,0.6)	(0.4,0.3)
$\mathbf{E_2}$	(0.5,0.2)	(0.2,0.8)	(0.3,0.3)	(0.2,0.5)	(0.8,0.3)
$\mathbf{E_3}$	(0.8,0.3)	(0.8,0.2)	(0.7,0.1)	(0.2,0.1)	(0.4,0.6)
$\mathbf{E_4}$	(0.5,0.5)	(0.2,0.1)	(0.5,0.4)	(0.5,0.1)	(0.4,0.3)
$\mathbf{E_5}$	(0.7,0.5)	(0.5,0.2)	(0.4,0.3)	(0.3,0.1)	(0.4,0.9)

Table 5: Final results

Proposed AoPs	$\mathbf{E_1}$	$\mathbf{E_2}$	$\mathbf{E_3}$	$\mathbf{E_4}$
	(0.441,0.2	(0.3665,0.2540)	(0.4144,0.4624)	(0.4846,0.3746)

Table 6: Ranking results

Proposed Method	$\boldsymbol{P_1}$	$\boldsymbol{P_2}$	$\boldsymbol{P_3}$	$\boldsymbol{P_4}$	**Ranking**
$\boldsymbol{F_c^h}$**SWA**	0.5983	0.5562	0.4750	0.5549	$P_1 > P_2 > P_4 > P_3$
$\boldsymbol{F_c^h}$**SOWA**	0.5826	0.5552	0.5069	0.5142	$P_1 > P_2 > P_4 > P_3$
$\boldsymbol{F_c^h}$**SHWA**	0.5247	0.5157	0.4262	0.4882	$P_1 > P_2 > P_4 > P_3$

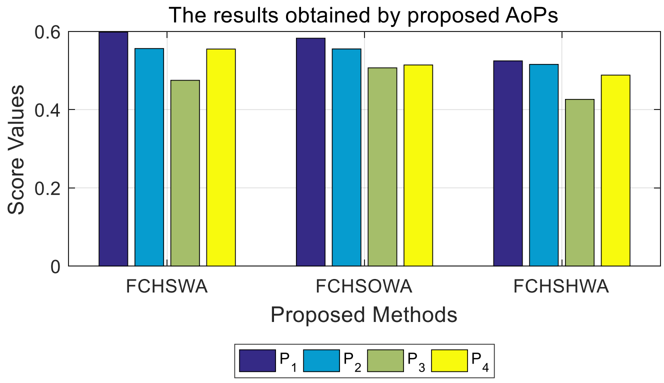

Figure 2: The results of proposed AoPs

The proposed model show that the P_1 is the best option which is clearly show in Figure 2 and Table 6.

6. Conclusions

This work introduced the F_c^hSHS idea, which is Characterized by a pair of fuzzy values that correspond to both the credibility of the fuzzy argument and the real environment of uncertainty and fuzziness, based on a fresh development of the fuzzy notion and then introduced Fuzzy credibility hyper soft set, average aggregation operators, basic notation of F_c^hSWA, F_c^hSOWA and F_c^hSHA operators. Next, a MADM strategy utilizing theF_c^hSWA, F_c^hSOWA and F_c^hSHA operator was created to address MADM issues in the context of FChSS. And finally, I introduced the table's ranking.

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