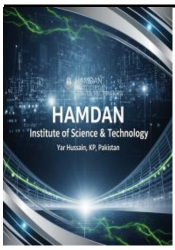

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Development of Multi-Criteria Decision Models using COPRAS Method through Pythagorean Cubic Fuzzy Numbers

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ABSTRACT

In this paper, we develop a novel decision-making model under the Pythagorean cubic fuzzy set (PCFS). PCFS prepare the larger space to explain the agreement, disagreement and hesitancy grades. Proportional assessment (COPRAS) methodology is one of the well-known multiple criteria group decision-making methods for solving the decision-making problems. In this paper, the COPRAS method develops for the decision-making method under a deterministic environment. Furthermore, to tackle the uncertainty of real-world projects and determine the critical path of projects by considering efficient criteria, a new group decision methodology is extended based on complex proportional assessment (COPRAS) method under Pythagorean cubic fuzzy sets PCFSs. In this respect, two numerical applications are considered to demonstrate the procedure of the proposed method. Finally, a comparative analysis and discussion with other method is provided. The proposed method is more effective and efficient as compared with existence methods for multi-criteria decision-making problems.

Keywords

Pythagorean cubic fuzzy sets
COPRAS method
Decision making model

1. *Introduction*

Multiple-criteria decision making (MCDM) plays an important role in the decision making theory and the DM theory divided into two main classes called multi objective and multi attribute decision making (MADM) [1]. Due the everyday use of criteria recognized with actual decision-making problems, MCDM approaches have become advanced in research area during the last decades [2-7]. The goal of MADM is to find an optimal alternative to having the highest degree of attraction with respect to all related degrees [8]. The contest of uncertainty in decision-making is the origin of such organization which is commonly an output of the unfamiliarity of decision makers. Under this circumstance, precise information is insufficient to model real problems. In answer to such difficulty and vagueness, some scientists propose the application of fuzzy sets (FS). The FS theory, introduced by Zadeh [9], is the generalization of the classic set theory showed the greater impact on real application in decision making. FS only considered the membership degree and some cases this theory failed to explain the uncertainty of an object. To cover this drawback, Atanassov [10] presented the notion of an intuitionistic fuzzy set (IFS) as the generalization of Zadeh's fuzzy sets, later extended it to an interval valued intuitionistic fuzzy set (IVIFS) [11]. The IFS and IVIFS have wide applications to analyze the problems with uncertainty, including questions related to MADM.

Li [12] proposed MADM method using the IFS, linear programming method [13], and TOPSIS method [14]. OWA aggregation operators were generalized in [15]. TOPSIS method was further extended to IVIFS in [16] for solving decision making problems. Yager [17] suggested a series of aggregation operations for PFSs. Yager and Abbasov [18] examined the relationship between Pythagorean membership degrees and complex numbers. Peng and Yang [19] extended Pythagorean fuzzy sets to interval-valued Pythagorean fuzzy sets and proposed some interval-valued Pythagorean fuzzy aggregation operators. A series of aggregation operators proposed by [20, 21] based on Pythagorean cubic fuzzy sets (PCFS). The Complex Proportional Assessment (COPRAS) technique was presented in [22] for MADM, which determine a solution and the ratio to the ideal solution to the worst ideal solution. This method was applied to many practical problems such as to evaluate a contractors based on 26 criteria [23] and analyzed the way of selecting an optimal building technological project. MADM method was further implemented in road construction [24]. In [25,26] well thought-out the application of a procedure for the multivariate design and multiple criteria analysis of the life cycle of a building based on COPRAS. In [27] used the COPRAS technique for assessing the market value in the analysis of construction and retrofit projects. The theoretical improvement of handling uncertainty in multi-criteria decision making process the COPRAS model is extended for such a problems using Pythagorean cubic fuzzy number (PCFN) method. The proposed framework for handling vagueness and uncertainty in the real-world evaluation and selection problems.

The aim of the paper is to extend the classical complex proportional assessment COPRAS method when both of alternatives on attributes and attributes weights are expressed by Pythagorean cubic fuzzy sets PCFSs. This paper is divided in six sections which is given as: The first section contains literature review and motivation of this paper. The preliminaries of the basic concept in section 2. The extended COPRAS method under the Pythagorean cubic fuzzy information discuss in section. The application of of the proposed method implemented in section 4. Further the comparison of the proposed method with existence method describe in 5. The conclusion of the paper is in section 6.

2. *Preliminaries*

This section provides the basic concept and fundamental result for the remaining paper.

Definition 1: [20] Pythagorean cubic fuzzy set A_{pc} is the set of ordered membership and non-membership function over a fixed universal set X as follows:

$$A_{pc}=\left\{\left\langle x,\mu_{A_{pc}}(x),\upsilon_{A_{pc}}(x)\right\rangle such\,that\; x\in X\right\} \tag{1}$$

where $\mu_{A_{pc}}(x)=\left\langle A_{pc}(x),\alpha_{pc}(x)\right\rangle$ the membership degree and $\upsilon_{A_{pc}}(x)=\left\langle \tilde{A}_{pc}(x),\beta_{pc}(x)\right\rangle$ is the non-membership degree under the condition that $0\le\left(\sup\left(A_{pc}(x)\right)\right)^2+\left(\sup\left(\tilde{A}_{pc}(x)\right)\right)^2\le 1$ and $0\le\alpha_{pc}^2(x)+\beta_{pc}^2\le 1$. Indeterminacy degree for Pythagorean cubic fuzzy set is given by:

$$\pi^*_{A_{pc}}=\left\langle\sqrt{1-\left(\sup\left(A_{pc}(x)\right)\right)^2-\left(\sup\left(\tilde{A}_{pc}(x)\right)\right)^2},\sqrt{1-\alpha_{pc}^2(x)-\beta_{pc}^2(x)}\right\rangle \tag{2}$$

Definition 2: [20] Let $A_{c1}=\left(\langle A_1,\alpha_1\rangle,\langle\tilde{A}_1,\beta_1\rangle\right)$, $A_{c2}=\left(\langle A_2,\alpha_2\rangle,\langle\tilde{A}_2,\beta_2\rangle\right)$ and $A_c=\left(\langle A,\alpha\rangle,\langle\tilde{A},\beta\rangle\right)$ are three Pythagorean cubic fuzzy numbers (PCFNs) and $\lambda>0$, where $A_1=[p_1,q_1],\ \tilde{A}_1=[\tilde{p}_1,\tilde{q}_1]$, $A_2=[p_2,q_2],\ \tilde{A}_2=[\tilde{p}_2,\tilde{q}_2]$ and $A=[p,q],\ \tilde{A}=[\tilde{p},\tilde{q}]$ the operation laws are given by.

1). $$A_{c1}\oplus A_{c2}=\left(\left\langle\left[\sqrt{p_1^2+p_2^2-p_1^2p_2^2},\sqrt{q_1^2+q_2^2-q_1^2q_2^2}\right],\sqrt{\alpha_1^2+\alpha_2^2-\alpha_1^2\alpha_2^2}\right\rangle,\left\langle[\tilde{p}_1\tilde{p}_2,\tilde{q}_1\tilde{q}_2],\beta_1\beta_2\right\rangle\right) \tag{3}$$

2). $$A_{c1}\otimes A_{c2}=\left(\left\langle[p_1p_2,q_1q_2],\alpha_1\alpha_2\right\rangle,\left\langle\left[\sqrt{\tilde{p}_1^2+\tilde{p}_2^2-\tilde{p}_1^2\tilde{p}_2^2},\sqrt{\tilde{q}_1^2+\tilde{q}_2^2-\tilde{q}_1^2\tilde{q}_2^2}\right],\sqrt{\beta_1^2+\beta_2^2-\beta_1^2\beta_2^2}\right\rangle\right) \tag{4}$$

3). $$\lambda A_{c1}=\left(\left\langle\left[\sqrt{1-\left(1-p_1^2\right)},\sqrt{1-\left(1-q_1^2\right)}\right],\sqrt{1-\left(1-\alpha_1^2\right)}\right\rangle,\left\langle\left[(\tilde{p}_1)^\lambda,(\tilde{q}_1)^\lambda\right],\beta^\lambda\right\rangle\right) \tag{5}$$

4. $$A_{c1}^\lambda=\left(\left\langle\left[p^\lambda,q^\lambda\right],\alpha^\lambda\right\rangle,\left\langle\left[\sqrt{1-\left(1-\tilde{p}_1^2\right)^\lambda},\sqrt{1-\left(1-\tilde{q}_1^2\right)^\lambda}\right],\sqrt{1-\left(1-\beta_1^2\right)^\lambda}\right\rangle\right) \tag{6}$$

5). $$A_{c1}^c=\left(\left\langle\tilde{A}_1,\pi_1\right\rangle,\left\langle A_1,\eta_1\right\rangle\right) \tag{7}$$

Definition 3: [20] Let $A_c=\left(\langle A,\alpha\rangle,\langle\tilde{A},\beta\rangle\right)$ where $A=[p,q]$ and $\tilde{A}=[\tilde{p},\tilde{q}]$ then the score function and accuracy degree of A_c are defined in (2) and (3).

$$Sc(A_c)=\left(\frac{p+q-\alpha}{3}\right)^2-\left(\frac{\tilde{p}+\tilde{q}-\beta}{3}\right)^2 \; where\; -1\le Sc(A_c)\le 1 \tag{8}$$

$$Ac\left(A_c\right)=\left(\frac{p+q-\alpha}{3}\right)+\left(\frac{\tilde{p}+\tilde{q}-\beta}{3}\right) \qquad where\, 0\le Ac\left(A_c\right)\le 1 \tag{9}$$

Definition 4: [20] Let A_{c1} and A_{c2} are two PCFNs then,

1. If $Sc(A_{c1})<Sc(A_{c2})$ then $A_{c1}<A_{c2}$.
2. If $Sc(A_{c1})=Sc(A_{c2})$ then,

 2.1. If $Ac(A_{c1})=Ac(A_{c2})$ then $A_{c1}=A_{c2}$

 2.2. If $Ac(A_{c1})<Ac(A_{c2})$ then $A_{c1}<A_{c2}$.

Definition 5: [20] Let $A_{ci}=\left(\langle A_i,\alpha_i\rangle,\langle\tilde{A}_i,\beta_i\rangle\right)$ where $A_i=[p_i,q_i]$ and $\tilde{A}_i=[\tilde{p}_i,\tilde{q}_i]$, $(i=1,2,\ldots n)$ are $'n'$ PCFNs and $w=w_1,w_2,\ldots w_n$ is the weight vector where $\sum_{i=1}^{n}w_i=1$ then Pythagorean cubic fuzzy weighted average operator (PCFWA operator) is defined as:

$$PCFWA(A_{ci})=\begin{pmatrix}\left\langle\left[\sqrt{1-\prod_{i=1}^{n}\left(1-p_i^2\right)^{w_i}},\sqrt{1-\prod_{i=1}^{n}\left(1-q_i^2\right)^{w_i}}\right];\sqrt{1-\prod_{i=1}^{n}\left(1-\alpha_i^2\right)^{w_i}}\right\rangle,\\ \left\langle\left[\prod_{i=1}^{n}\tilde{p}_i^{w_i},\prod_{i=1}^{n}\tilde{q}_i^{w_i}\right];\prod_{i=1}^{n}\beta_i^{w_i}\right\rangle\end{pmatrix} \tag{10}$$

And Pythagorean cubic fuzzy weighted geometric operator is defined as:

$$PCFWG(A_{ci})=\begin{pmatrix}\left\langle\left[\prod_{i=1}^{n}p_i^{w_i},\prod_{i=1}^{n}q_i^{w_i}\right];\prod_{i=1}^{n}\alpha_i^{w_i}\right\rangle,\\ \left\langle\left[\sqrt{1-\prod_{i=1}^{n}\left(1-\tilde{p}_i^2\right)^{w_i}},\sqrt{1-\prod_{i=1}^{n}\left(1-\tilde{q}_i^2\right)^{w_i}}\right];\sqrt{1-\prod_{i=1}^{n}\left(1-\beta_i^2\right)^{w_i}}\right\rangle\end{pmatrix} \tag{11}$$

3. *COPRAS Method with Pythagorean Cubic Fuzzy Information*

In this section, the extended COPRAS method under the Pythagorean cubic fuzzy information developed. This method is more accurate and efficient other than method for decision support systems.

Suppose $'m'$ number of alternatives $A_i : i=(1,2,\ldots m)$ are invalid in the decision-making problem with $'n'$ number of criteria $C_j : (j=1,2,\ldots,n)$ for evaluating these alternatives. Suppose that there are $'K'$ number of exports in the decision-making process. Also the attribute weights are not determined but expressed by Pythagorean cubic fuzzy numbers (PCFNs). This group decision-making process has the following steps.

Step 1: In this step determine the importance of decision makers. Suppose that $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_K)$ is the vector indicating the importance of the decision makers, where $\lambda_k : k=1,2,\ldots,K$ and $\lambda_k > 0$ is the importance of k^{th} decision maker. Note that $\sum_{k=1}^{K}\lambda_k=1$ and if all importance are similar, Then, $\lambda_1=\lambda_2=\ldots=\lambda_K=\frac{1}{K}$

Step 2: Each export gives his/her individual evaluation regarding the rating of on the attribute and attribute weight. Prepare the decision matrix. Each entry of the decision matrix is expressed in the form of Pythagorean cubic number. Suppose that $\rho_{ij}^k\ (i=1,2,\ldots m;\ j=1,2,\ldots,n)$ is the k^{th} experts evaluation of the alternative A_i then $\rho_{ij}^k=\left(\left\langle\left[p_{ij}^k,q_{ij}^k\right];\alpha_{ij}^k\right\rangle,\left\langle\left[\tilde{p}_{ij}^k,\tilde{q}_{ij}^k\right];\beta_{ij}^k\right\rangle\right)$. Finally, the decision matrix is constructed as follows:

$$\widehat{X}_{ij}^{k}=\begin{bmatrix}\rho_{11}^k & \rho_{12}^k & \cdots & \rho_{1n}^k\\ \rho_{21}^k & \rho_{22}^k & \cdots & \rho_{2n}^k\\ \vdots & \vdots & \ddots & \vdots\\ \rho_{m1}^k & \rho_{m2}^k & \cdots & \rho_{mn}^k\end{bmatrix} \tag{12}$$

Step 3: On this step determine the weights of criteria. Suppose that j expressed by decision makers the aggregated weight of criterion $j, w_j=\left(\left\langle\left[p_j,q_j\right];\alpha_j\right\rangle,\left\langle\left[\tilde{p}_j,\tilde{q}_j\right],\beta_j\right\rangle\right)$ then, the weights of criteria will be computed as follows:

$$\begin{aligned}&w_j=PCFWA_{\delta}(w_j^1,w_j^2,\ldots,w_j^K)\\&=\begin{pmatrix}\left\langle\left[\left(\sqrt{1-\prod_{k=1}^{K}\left(1-\left(p_j^k\right)^2\right)^{\lambda_k}}\right),\left(\sqrt{1-\prod_{k=1}^{K}\left(1-\left(q_j^k\right)^2\right)^{\lambda_k}}\right)\right],\sqrt{1-\prod_{k=1}^{K}\left(1-\left(\alpha_j^k\right)^2\right)^{\lambda_k}}\right\rangle,\\ \left\langle\left[\prod_{k=1}^{K}\left(p_j^k\right)^{\lambda_k},\prod_{k=1}^{K}\left(q_j^k\right)^{\lambda_k}\right],\prod_{k=1}^{K}\left(\beta_j^k\right)^{\lambda_j}\right\rangle\end{pmatrix}\end{aligned} \tag{13}$$

Step 4: Applying Pythagorean cubic fuzzy weighted average operator (PCFWA operator) to the elements of the individual decision matrices to construct the aggregated decision matrix. The aggregated decision matrix will be as follows:

$$\widehat{X}^{k}=\begin{bmatrix}\rho_{11} & \rho_{12} & \cdots & \rho_{1n}\\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \rho_{m1} & \rho_{m2} & \cdots & \rho_{mn}\end{bmatrix} \tag{14}$$

Where ρ_{ij} is calculated as follows:

$$\begin{aligned}&\rho_{ij}=PCFWA_{\lambda}(\rho_{ij}^1,\rho_{ij}^2,\ldots,\rho_{ij}^K)\\&=\begin{pmatrix}\left\langle\left[\sqrt{1-\prod_{k=1}^{K}\left(1-\left(p_{ij}^k\right)^2\right)^{\lambda_k}},\sqrt{1-\prod_{k=1}^{K}\left(1-\left(q_{ij}^k\right)^2\right)^{\lambda_k}}\right]\sqrt{1-\prod_{k=1}^{K}\left(1-\left(\alpha_{ij}^k\right)^2\right)^{\lambda_k}}\right\rangle,\\ \left\langle\left[\prod_{k=1}^{K}\left(\tilde{p}_{ij}^k\right)^{\lambda_k},\prod_{k=1}^{K}\left(q_{ij}^k\right)^{\lambda_k}\right],\prod_{k=1}^{K}\left(\beta_{ij}^k\right)^{\lambda_k}\right\rangle\end{pmatrix}\end{aligned} \tag{15}$$

Step 5: Calculate the normalized decision matrix. The weighted matrix is calculated as $\hat{\hat{X}} = [\hat{\rho}_{ij}],\ (i = 1, 2, .., n\ and\ j = 1, 2, ..., m)$ where $\hat{\rho}_{ij} = w_j . \rho_{ij}$ and hence

$$\hat{\rho}_{ij} = \begin{pmatrix} \langle [p_{ij}p_j, q_{ij}q_j], \alpha_{ij}\alpha_j \rangle, \\ \langle \left[\sqrt{\tilde{p}_{ij}^2 + \tilde{p}_j^2 - \tilde{p}_{ij}^2\tilde{p}_j^2}, \sqrt{\tilde{q}_{ij}^2 + \tilde{q}_j^2 - \tilde{q}_{ij}^2\tilde{q}_j^2}\right], \sqrt{\beta^2{}_{ij} + \beta^2{}_j - \beta^2{}_{ij}\beta^2{}_j} \rangle \end{pmatrix} \tag{16}$$

Step 6: For the benefit sum all the values of criteria. Let $B = \{1, 2, ...l\}$ be the set of criteria the higher values of which are bitter. For each alternative calculate the following index.

$$P_i = \sum_{j \in B} \hat{\rho}_{ij} \tag{17}$$

Step 7: Sum the values of cost criteria. Let $B^{\square} = \{l+1, l+2, ..., n\}$ be the set of criteria the lower values of which are better. Then, for each alternative calculate the following index.

$$R_i = \sum_{j \in B^{\square}} \hat{\rho}_{ij} \tag{18}$$

Step 8: Determine the minimal value of R_i such that $R_{\min} = \min_i (R_i)\ where\ i = 1, 2, ..., m$ ·

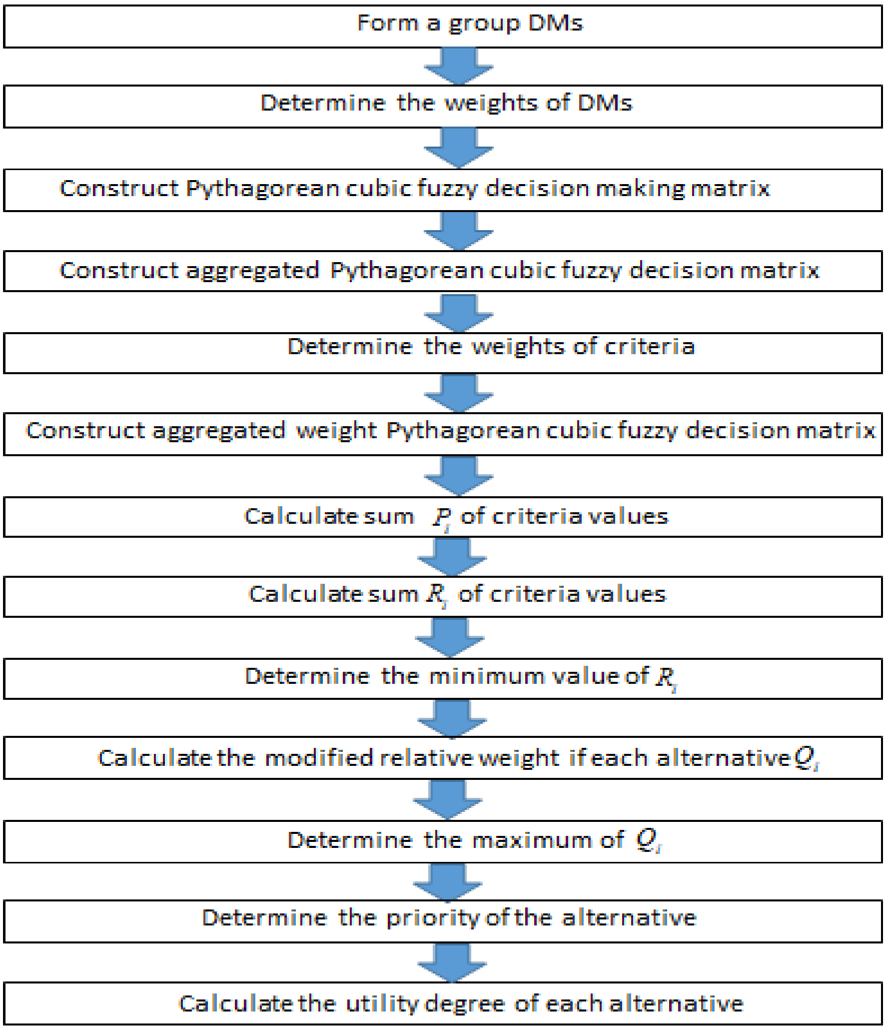

Figure 1: Step-by-step procedure of proposed method for the selection problems.

Step 9: Determine the relative weight of each alternative denoted by Q_i such that

$$Q_i = Sc(P_i) + \frac{Sc(R_{\min}) \sum_{i=1}^{m} Sc(R_i)}{Sc(R_i) \sum_{i=1}^{m} \left(\frac{Sc(R_{\min})}{Sc(R_i)} \right)}$$

$$= Sc(P_i) + \frac{\sum_{i=1}^{m} Sc(R_i)}{\sum_{i=1}^{m} \frac{1}{Sc(R_i)}} \tag{19}$$

In (12) $Sc(R_i)$ and $Sc(P_i)$ are the score functions of R_i and P_i respectively.

Step 10. Calculate optimality criterion $Q_{\max}$ as follows:

$$Q_{\max} = \max(Q_i) \ \ where \ (i = 1, 2, ..., m) \tag{20}$$

Step 11: The degree of utility for each alternative id calculated by comparing the other alternative with the best alternative. The values of the degree of utility rang from 0% to 100% between the worst and the best alternative. The utility degree N_j for each alternative j is determined as follows.

$$N_i = \frac{Q_i}{Q_{\max}} \times 100\% \tag{21}$$

4. *Illustrative examples*

In this section we are going to give a brief illustrative example of the new approach in an investment strategy decision-making process. Investment decision section is most important in business. Most of companies want to invest money and expect that it will have a sufficient efficient effect on the company in the long term. Before implementing the project, it is necessary for the scientific investment decision-making.

*4.1. **Example 1**.*

Assume that a company wants to invest a large sum of money in a region and they consider the following four investment alternative.

A_1 (Invest in the African Market), A_2 (Invest in the American Market), A_3 (Invest in the Asian Market) and A_4 (Invest in the European Market) based on four attributes: C_1 (Risk analysis), C_2 (Benefit in the short term), C_3 (Benefit in the long term) and C_4 (Socio-political issues). All the detailed description of the problem is provided as in PCFNs. The fund manager determines attribute weights as $w_1 = 0.29,\ w_2 = 0.13,\ w_3 = 0.32$ and $w_4 = 0.26$. The provided decision matrix is as follows:

$$X = \begin{matrix} \\ A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \end{matrix} \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \\ \begin{pmatrix} \langle [0.4, 0.5]; 0.4 \rangle, \\ \langle [0.5, 0.6]; 0.5 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.3, 0.6]; 0.4 \rangle \\ \langle [0.2, 0.4]; 0.6 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.2, 0.3]; 0.2 \rangle, \\ \langle [0.3, 0.5]; 0.5 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.6, 0.7]; 0.5 \rangle, \\ \langle [0.5, 0.7]; 0.6 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle [0.3, 0.5]; 0.4 \rangle, \\ \langle [0.3, 0.6]; 0.6 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.4, 0.5]; 0.4 \rangle, \\ \langle [0.5, 0.7]; 0.6 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.5, 0.6]; 0.5 \rangle, \\ \langle [0.4, 0.7]; 0.6 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.4, 0.7]; 0.5 \rangle, \\ \langle [0.3, 0.4]; 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle [0.5, 0.6]; 0.3 \rangle \\ \langle [0.3, 0.7]; 0.4 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.5, 0.7]; 0.5 \rangle, \\ \langle [0.6, 0.7]; 0.6 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.6, 0.7]; 0.2 \rangle, \\ \langle [0.4, 0.6]; 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.4, 0.6]; 0.1 \rangle, \\ \langle [0.3, 0.5]; 0.3 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle [0.2, 0.4]; 0.6 \rangle, \\ \langle [0.4, 0.7]; 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.4, 0.6]; 0.2 \rangle, \\ \langle [0.3, 0.7]; 0.4 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.6, 0.7]; 0.5 \rangle, \\ \langle [0.3, 0.6]; 0.6 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.4, 0.5]; 0.4 \rangle, \\ \langle [0.3, 0.5]; 0.6 \rangle \end{pmatrix} \end{bmatrix}$$

$$\hat{X} = \begin{bmatrix} \begin{pmatrix} \langle [0.2220,0.2829];0.2220 \rangle, \\ \langle [0.8179,0.8623];0.6270 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.1104,0.2374];0.1497 \rangle, \\ \langle [0.8112,0.8877];0.9357 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.1139,0.0934];0.1139 \rangle, \\ \langle [0.5975,0.7459];0.8011 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.3310,0.4008];0.2685 \rangle, \\ \langle [0.8351,0.9114];0.8756 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle [0.1643,0.2829];0.2220 \rangle, \\ \langle [0.7053,0.8623];0.8623 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.1497,0.1916];0.1497 \rangle, \\ \langle [0.9138,0.9547];0.9357 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.2966,0.3648];0.2966 \rangle, \\ \langle [0.7459,0.8921];0.8492 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.2105,0.4008];0.2685 \rangle, \\ \langle 0[.7312,0.7880];0.9114 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle [0.2829;0.3484];0.1643 \rangle, \\ \langle [0.7053,0.9017];0.7667 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.1916,0.2895];0.1916 \rangle, \\ \langle [0.9357,0.9547];0.9357 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.3648,0.4403];0.1139 \rangle, \\ \langle [0.7459,0.8492];0.8921 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.2105,0.3310];0.0511 \rangle, \\ \langle [0.7312,0.8351];0.0081 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle [0.1085,0.2220];0.3484 \rangle, \\ \langle [0.7667,0.9017];0.9017 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.1497,0.2374];0.0728 \rangle, \\ \langle [0.8551,0.9547];0.8877 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.3648,0.3648];0.2966 \rangle, \\ \langle [0.6803,0.8492];0.8492 \rangle \end{pmatrix} & \begin{pmatrix} \langle [0.2105,0.2685];0.2105 \rangle, \\ \langle [0.7312,0.8351];0.8756 \rangle \end{pmatrix} \end{bmatrix}$$

We suppose that C_2, C_3 and C_3 as criteria benefit while C_1 as the cost criterion. For this condition the values of P_i, R_i and Q_i are follows:

Table 1: The obtained values of P_i, Q_i and R_i. Also the rank of each alternative is given in table.

Alternative	P_i	R_i	Q_i	Rank

A_1	$\begin{pmatrix}\langle[0.3631,0.4635];0.3235\rangle,\\ \langle[0.4048,0.6035];0.6563\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2220,0.2829];0.2220\rangle,\\ \langle[0.8179,0.8623];0.6270\rangle\end{pmatrix}$	-0.0042	2
A_2	$\begin{pmatrix}\langle[0.3846,0.5469];0.4156\rangle,\\ \langle[0.4984\quad 0.6711];0.7242\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1643,0.2829];0.2220\rangle,\\ \langle[0.7053,0.8623];0.8623\rangle\end{pmatrix}$	-0.0110	3
A_3	$\begin{pmatrix}\langle[0.4493,0.5851];0.2274\rangle,\\ \langle[0.5103,0.6770];0.0068\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2829,0.3484];0.1643\rangle,\\ \langle[0.7053,0.9017];0.7667\rangle\end{pmatrix}$	-0.1010	4
A_4	$\begin{pmatrix}\langle[0.4360,0.5211];0.3647\rangle,\\ \langle[0.3126,0.6770];0.6601\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1085,0.2220];0.3484\rangle,\\ \langle[0.7667,0.9017];0.9017\rangle\end{pmatrix}$	0.0084	1

The obtained rank by the proposed method is $A_4 > A_1 > A_3 > A_2$. Note that decision makers can be considered the membership function as a satisfaction degree and non-membership function as a dissatisfaction degree. In fact, the evaluations can make in the form of a satisfaction and dissatisfaction by the decision makers as PFSs.

Table 2: Comparison of proposed method with PCFWA and PCFWG.

Methods	A_1	A_2	A_3	A_4	Ranking
PCFWA[20]	0.0198	0.0229	0.0533	0.0598	$A_4 > A_3 > A_2 > A_1$
PCFWG[20]	0.0210	0.0331	0.0351	0.0383	$A_4 > A_3 > A_2 > A_1$
Proposed method	-0.0042	-0.0110	-0.1010	0.0084	$A_4 > A_1 > A_3 > A_2$

4.2. ***Example 2***.

Suppose that a company wants to contract its yearly maintenance operation. From the external maintenance service providers (MSP) received four proposals to the company. For the MPS, the company established a team which consisting of three members. All of these members have equal importance, i.e., $\lambda = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Also, four criteria are considered C_1 (proposed price), C_2 (duration of maintenance), C_3 (reputation) and C_4 (specialty of workers). The following steps are involving in the decision-making process.

Each decision maker preforms their evaluations of alternatives. The scale is presented in the following table.

Table 3. linguistic term in it values in the form of Pythagorean cubic fuzzy (PCFN).

Linguistic term	PCFNs
Extremely good (EG)	$(\langle[0.9,1];0\rangle,\langle[0,0.1];0.1\rangle)$
Very-very good (VVG)	$(\langle[0.8,0.9];0.1\rangle,\langle[0.11,0.15];0.1\rangle)$
Very good (VG)	$(\langle[0.81,0.85];0.25\rangle,\langle[0.25,0.29];0.3\rangle)$
Good (G)	$(\langle[0.7,0.8];0.25\rangle,\langle[0.3,0.35];0.3\rangle)$
Medium good (MG)	$(\langle[0.49,0.69];0.3\rangle,\langle[0.36,0.4];0.4\rangle)$
Fair (F)	$(\langle[0.5,0.55];0.5\rangle,\langle[0.41,0.49];0.45\rangle)$
Medium bad (MB)	$(\langle[0.3,0.4];0.55\rangle,\langle[0.5,0.6];0.5\rangle)$
Bad (B)	$(\langle[0.2,0.3];0.6\rangle,\langle[0.7,0.79];0.6\rangle)$
Very bad (VB)	$(\langle[0.1,0.2];0.7\rangle,\langle[0.8,0.85];0.7\rangle)$
Very-very bad (VVB)	$(\langle[0,0.1];0.8\rangle,\langle[0.86,0.9];0.8\rangle)$

Individual linguistic term decision matrices by DM1, DM2 and DM3 are:

Table 4: The linguistic term matrices express by three decision makers DM1, DM2 and DM3.

DM1	C_1	C_2	C_3	C_4	DM2	C_1	C_2	C_3	C_4	DM3	C_1	C_2	C_3	C_4
A_1	MG	F	B	G	A_1	F	MB	MG	G	A_1	G	MG	MB	VG
A_2	G	F	G	MG	A_2	MG	G	G	G	A_2	VG	F	G	G

A_3	F	G	MG	VG	A_3	MG	VG	G	MG	A_3	G	MG	MG	MG
A_4	F	MG	F	MG	A_4	F	F	MB	MB	A_4	MG	F	F	F

Decision makers also express their judgments with regard to the importance of criteria for the evaluating alternatives. The preference of decision makers for the weight of criteria in linguistic terms is given by:

Table 5: Linguistic terms and its values in the form of PCFNs.

In linguistic terms	In Pythagorean cubic fuzzy numbers (PCFNs)
Very important (VI)	$(\langle[0.8,0.9];0.1\rangle,\langle[0.1,0.2];0.1\rangle)$
Important (I)	$(\langle[0.7,0.8];0.2\rangle,\langle[0.2,0.3];0.3\rangle)$
Medium (M)	$(\langle[0.6,0.7];0.1\rangle,\langle[0.3,0.4];0.5\rangle)$
Unimportant (U)	$(\langle[0.5,0.55];0.3\rangle,\langle[0.5,0.6];0.1\rangle)$
Very unimportant	$(\langle[0,0.4];0.6\rangle,\langle[0.6,0.7];0.4\rangle)$

Table 6: Individual decision matrices.

Decision makers/ criteria	C_1	C_2	C_3	C_4
DM1	M	M	I	I
DM2	I	I	M	I
DM3	I	M	U	I

Based on (13) the aggregated weights and importance vector λ found by decision makers are calculated as follows.

$$w_1=(\langle[0.8184\quad 0.8781];0.4099\rangle,\langle[0.2289,0.3302];0.3557\rangle),$$

$$w_2=(\langle[0.7979,0.8590];0.3670\rangle,\langle[0.2621,0.3634];0.4217\rangle),$$

$$w_3=\langle[0.7801\quad 0.8367];0.4519\rangle,\langle[0.3107,0.4160];0.2466\rangle \text{ and}$$

$$w_4=(\langle[0.8367\quad 0.8944];0.4472\rangle,\langle[0.2000,0.3000];0.3000\rangle)$$

$$A=\begin{bmatrix} \begin{pmatrix}\langle[0.5810,0.7010];0.3719\rangle\\ \langle[0.3538,0.4094];0.3780\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.4434,0.5704];0.4682\rangle,\\ \langle[0.4195,0.4899];0.4481\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.3574,0.5127];0.5093\rangle,\\ \langle[0.5013,0.5745];0.4932\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.7435,0.8185];0.2500\rangle,\\ \langle[0.2823,0.3287];0.3000\rangle\end{pmatrix} \\ \begin{pmatrix}\langle[0.6994,0.7911];0.2678\rangle,\\ \langle[0.3000,0.3437];0.3302\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.5835,0.6637];0.4383\rangle,\\ \langle[0.3695,0.4380];0.3931\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.7000,0.8000];0.2500\rangle,\\ \langle[0.3000,0.3500\ ];0.3000\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.6461,0.7694];0.2680\rangle,\\ \langle[0.3188,0.3659];0.3302\rangle\end{pmatrix} \\ \begin{pmatrix}\langle[0.5810,0.7010];0.3719\rangle,\\ \langle[0.3538,0.4094];0.3780\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.6994,0.7911];0.2680\rangle,\\ \langle[0.3000,0.3437];0.3302\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.5785,0.7333];0.2844\rangle,\\ \langle[0.3388,0.3826];0.3634\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.6454,0.7590];0.2844\rangle,\\ \langle[0.3188\quad 0.3593];0.3634\rangle\end{pmatrix} \\ \begin{pmatrix}\langle[0.4967;0.6050];0.4473\rangle,\\ \langle[0.3926,0.4579];0.4327\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.4967,0.6050];0.4473\rangle,\\ \langle[0.3926,0.4579];0.4327\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.4473,0.5078];0.5176\rangle,\\ \langle[0.4380,0.5242];0.4661\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.4434,0.5705];0.4682\rangle,\\ \langle[0.4195,0.4899];0.4481\rangle\end{pmatrix} \end{bmatrix}$$

Weighted matrix based on (16) as follows:

$$\hat{A}=\begin{bmatrix} \begin{pmatrix}\langle[0.4755,0.6155];0.1524\rangle,\\ \langle[0.4135,0.5083];0.5013\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.3538,0.4900];0.1718\rangle,\\ \langle[0.4823,0.5834];0.5856\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.2788,0.4290];0.2302\rangle,\\ \langle[0.5688,0.6678];0.5378\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.6221,0.7321];0.1118\rangle,\\ \langle[0.3413,0.4340];0.4146\rangle\end{pmatrix} \\ \begin{pmatrix}\langle[0.5724,0.6947];0.1098\rangle,\\ \langle[0.3711,0.4629];0.4709\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.4656,0.5701];0.1609\rangle,\\ \langle[0.4425,0.5464];0.5522\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.5461,0.6694];0.1130\rangle,\\ \langle[0.4217,0.5238];0.3812\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.5406,0.6882];0.1198\rangle,\\ \langle[0.3709,0.4603];0.4350\rangle\end{pmatrix} \\ \begin{pmatrix}\langle[0.4755,0.6155];0.1524\rangle,\\ \langle[0.4135,0.5083];0.5013\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.5581,0.6796];0.0984\rangle,\\ \langle[0.3905,0.4843];0.5172\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.4513,0.6136];0.1285\rangle,\\ \langle[0.4475,0.5423];0.4299\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.5316,0.6788];0.1272\rangle,\\ \langle[0.3709,0.4555];0.4584\rangle\end{pmatrix} \\ \begin{pmatrix}\langle[0.4065,0.5313];0.1833\rangle,\\ \langle[0.4455,0.5439];0.5386\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.3963,0.5197];0.1642\rangle,\\ \langle[0.4607,0.5604];0.5760\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.3489,0.4249];0.2339\rangle,\\ \langle[0.5195,0.6327];0.5146\rangle\end{pmatrix} & \begin{pmatrix}\langle[0.3710,0.5103];0.2094\rangle,\\ \langle[0.4571,0.5553];0.5222\rangle\end{pmatrix} \end{bmatrix}$$

In our case C_1 and C_2 are cost criteria while C_3 and C_4 are criteria for the benefit. The computed values of P_i, Q_i and R_i are shown in the following table.

Table 7: The obtained values of P_i, Q_i and R_i. Also the rank of each alternative is given in table.

Alternatives	P_i	R_i	Q_i	**Rank**
A_1	$\begin{pmatrix}\langle[0.6593,0.7883];0.2546\rangle,\\ \langle[0.1941,0.2898];0.2230\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.5683,0.7266];0.2282\rangle,\\ \langle[0.1994,0.2965];0.2937\rangle\end{pmatrix}$	0.3206	3
A_2	$\begin{pmatrix}\langle[0.7094,0.8423];0.1641\rangle,\\ \langle[0.1564,0.2411];0.1658\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.6880,0.8067];.1940\rangle,\\ \langle[0.1642,0.2529];0.2600\rangle\end{pmatrix}$	0.3780	1
A_3	$\begin{pmatrix}\langle[0.6548,0.8147];0.1801\rangle,\\ \langle[0.1660,0.2470];0.1971\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.6835,0.8159];0.1808\rangle,\\ \langle[0.1654,0.2462];0.2593\rangle\end{pmatrix}$	0.3496	2
A_4	$\begin{pmatrix}\langle[0.4926,0.6276];0.3101\rangle,\\ \langle[0.2375,0.3513];0.2687\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.5444,0.6900];0.2442\rangle,\\ \langle[0.2052,0.3048];0.3103\rangle\end{pmatrix}$	0.2316	4

Therefore, the finial raking of alternatives is $A_2 > A_3 > A_1 > A_4$.

5. *Comparison analysis*

In this section, it has been concluded that the best alternative coincides with the existing approaches results and proposed approach results is more pessimistic than existing methodology values for taking a decision. From this following table, it is been observed that the nature of the relative score follows the same trend.

Table 8: Comparison of proposed method with different techniques

Techniques	**A₁**	**A₂**	**A₃**	**A₄**	**Ranking**
PCFWA [20]	0.2089	0.2685	0.2576	0.1393	$A_2 > A_3 > A_1 > A_4$
PCFWG [20]	-0.2036	-0.2001	-0.2021	-0.2131	$A_2 > A_3 > A_1 > A_4$
Proposed method	0.3206	0.3780	0.3496	0.2316	$A_2 > A_3 > A_1 > A_4$

6. *Conclusions*

Decision making is an important tool to solving practical problems that's requires judgments of decision makers about the importance of a set of criteria and alternatives. Fuzzy set theory is appropriate way to deal multi-criteria copes with insufficient and uncertain information in complex group decision making. Being a generalization of interval valued Pythagorean fuzzy set, the Pythagorean cubic fuzzy set PCFS allows experts or decision makers to prepare an additional possibility to represent imperfect knowledge. This study presents the extended form of COPRAS method. This method is more adequate to deal with uncertainty than traditional approaches in the manufacturing industry. In this method, the ratings of each alternative with respect to each criterion and weights of criteria are linguistic terms as characterized by Pythagorean cubic fuzzy numbers. The application of the proposed method is examined in two numerical examples. Moreover, a comparative analysis was performed between proposed method and the Pythagorean cubic fuzzy group method. The suggested method can be applied with various selection and ranking problems in the different fields as a common uncertain decision analysis method.

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